# Resource Curse or Blessing? Sovereign Risk in Resource-Rich Emerging Economies

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**Abstract**

Business cycles in oil-exporting emerging economies are tied to oil price fluctuations. Many of these economies also have large external debt ratios and have defaulted since the 1980s. We show that, in addition, higher oil prices and/or oil production are associated with lower sovereign risk in the long-run while the opposite is true for higher oil reserves. We propose a model of sovereign default and oil extraction consistent with these observations. The default payoff is endogenous and depends on oil reserves. Higher oil production or prices reduce country risk by increasing debt repayment capacity but larger reserves can increase it by making autarky more valuable. Without default risk, the model is akin to an RBC model with terms-of-trade shocks: High oil prices incentivize increasing oil production and reducing reserves so that the gross return of oil equals the world interest rate plus a standard "equity premium." In contrast, with default risk, the sovereign internalizes that higher reserves reduce the price of its debt because of the higher option value of default and this increases the sovereign's rate of return on oil.

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## 1 Introduction

Fluctuations in commodity prices are a key determinant of macroeconomic performance in resource-rich economies. This is particularly the case in oil-exporting countries. Well-known transmission mechanisms connect world oil-price fluctuations with macroeconomic outcomes via their effects on incentives to extract oil and adjust oil reserves and on incentives to consume, invest in physical capital, and borrow or lend in international financial assets.[1] In contrast, the relationship between oil prices, default risk, and macroeconomic dynamics has been studied much less, despite the fact that country risk itself is also a well-known determinant of business cycles in emerging markets (see Uribe & Yue (2006); E. G. Mendoza & Yue (2012)) and that oil revenues are a significant driver of government solvency and country risk in oil-exporting countries.

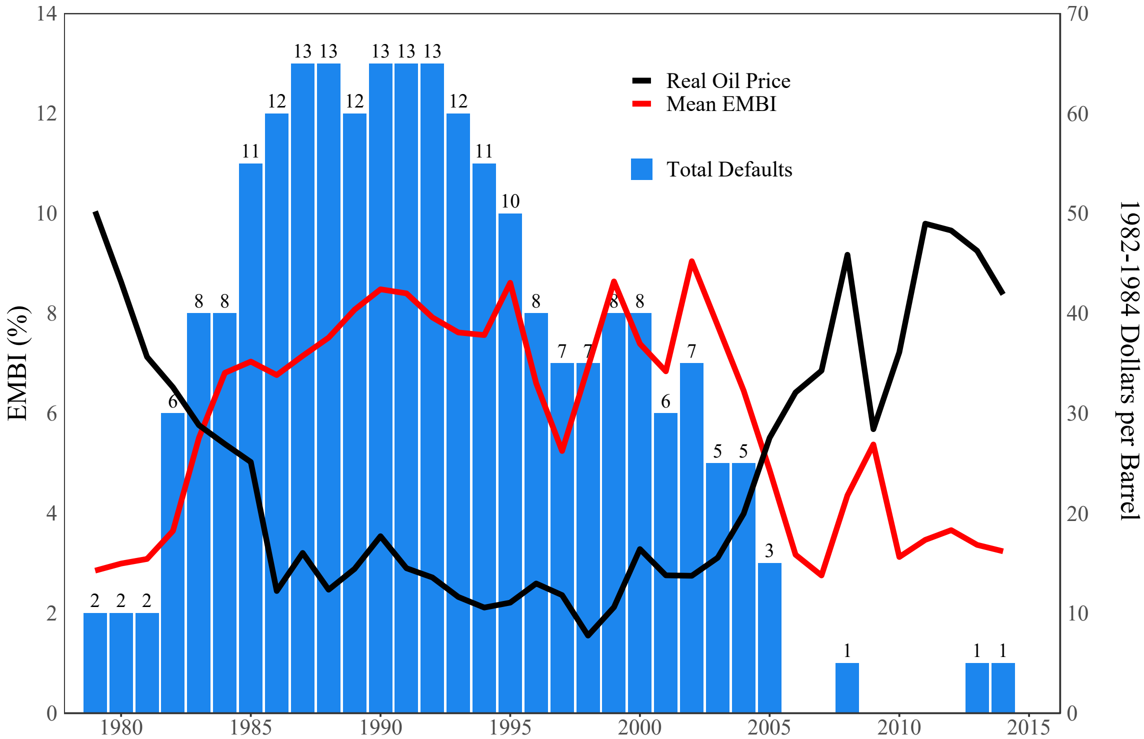
Figure 1 is indicative of the potential relevance of the relationship between oil prices, default risk and default events in countries classified as oil-exporting emerging economies.[2] This Figure shows the total number of default events observed in these countries by year from 1979 to 2015 (top of bars), the mean of the Emerging Market Bond Index (EMBI) for these countries (left axis) and the real price of oil over the same years (right axis). It is evident from this plot that both country risk and the number of default events rise (fall) sharply as the price of oil falls (rises). The 1980-2005 period illustrates these co-movements over the medium term, but they are observable even in the short run (for instance, while between 2005 and 2007 the price is high, there are no defaults and country risk goes down, as soon as the price drops in 2008 default risk goes up and a default event occurs).

In this paper, we study the empirical regularities connecting oil prices, oil production, and oil reserves to country risk and propose a new model that can rationalize those regularities. We start by examining the main stylized facts about the relationship between oil prices, sovereign risk and macroeconomic performance of oil-exporting economies. We show that, even though countries that produce more oil tend to display lower sovereign risk, the opposite is true for countries that have more oil reserves. Using dynamic fixed effects estimation,

[1]These issues have been widely studied in the international business cycle literature, particularly the branch focusing on terms-of-trade shocks (e.g. Backus et al. (1994); E. Mendoza (1995); Ben Zeev et al. (2017); Schmitt-Grohé & Uribe (2018)

[2]We include the thirty largest oil producing emerging economies as of 2010. See Appendix A for the exact list of countries and data sources.

Figure 1: Oil Prices and Default Episodes




we find that the short-run elasticity of country risk with respect to changes in oil production is around $0.05\%$ and the long-run elasticity with respect to oil reserves is around $-0.15\%$. That is, when oil production increases by $1\%$ country risk decreases by $0.05\%$, and when oil reserves increase by $1\%$, country risk increases by $0.16\%$.

We then develop a model of sovereign default on external debt in which optimal plans regarding oil extraction, debt, oil reserves, and default interact. We derive analytic results relating oil-price shocks to default incentives and default risk and conduct a quantitative analysis to assess the model's ability to explain the empirical facts.

Examining data for the 30 largest emerging market oil producers over the period 1979-2014, we found that these countries hold an average external public debt to GDP ratio of roughly $29\%$ and sixteen countries in the sample have experienced between one and five default episodes.[3] We highlight three features of the relationship between country risk and the

[3]We use external public debt data from the World Bank where public and publicly guaranteed debt comprises long-term external obligations of public debtors, including the national government, political subdivisions (or an agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed

size of the oil sector: First, as is natural to expect, a given oil exporting country is perceived by investors as less risky, the higher their oil production and the higher oil prices, allowing its public sector to support higher levels of public debt. Second, and perhaps less natural to expect, in the long run, country risk perception increases the higher the level of oil reserves of the country. This may reflect the fact that having a large stock of oil increases a country's outside option (the value of autarky), making default more appealing. Third, the data also shows that during most default episodes, the median oil exporting country increases net oil exports. This evidence suggests that a country in default and excluded from international financial markets, increases its oil exports to withstand the consequences of financial autarky.

When we explore the relationship between oil price changes and macro performance, we find that increases in oil prices are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price decreases are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and a higher number of default events.

We build a small open economy model with two types of goods: a tradable and non-storable consumption good and oil. The sovereign government owns all oil reserves—and makes all decisions regarding its extraction—and can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets but cannot commit to repaying its debt. The relative price of oil and the consumption good are exogenously given.

We find that theory predicts that long-run reserves of the resource have two opposing effects in determining the long-term sovereign risk premium. Higher stock of reserves allow the country to have a higher extraction rate to support debt repayments, lowering default risk. However, they also allow the country to use the resource during default times, making the value of default more attractive. The tension between these two forces determines the long run default risk premium.

Our work links two large strands of the literature, one related to sovereign risk and another related to commodity markets. In our model, a sovereign facing incomplete financial markets may find optimal to default, as in Eaton & Gersovitz (1981); Aguiar & Gopinath (2006); Arellano (2008), and most of the literature thereafter. The sovereign however, owns

for repayment by a public entity. Data are in current local currency units.

a stock of reserves of a commodity that can (in addition to foreign borrowing) be used to smooth consumption affecting in turn the default risk premium. Bouri et al. (2017) document the transmission of volatility from commodity markets to credit default swaps (CDS) spreads of emerging markets. They find significant volatility spillovers specially coming from energy and precious metals. Reinhart et al. (2016) document how major spikes in sovereign defaults occur when capital inflows surge and are followed by busts in capital and commodity markets. Fernández et al. (2017) present an empirical framework in which multiple commodity prices transmit to domestic business cycles, explaining up to 33% of output fluctuations of individual countries.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the calibration and quantitative analysis of the model, and Section 5 concludes.

## 2 Empirical Evidence

This section documents important empirical regularities linking oil prices, oil production, oil reserves, and sovereign risk for the 1979-2016 period. We start by describing the data and then move on to document the stylized facts.

### 2.1 Data

We collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, oil prices, total public debt, total external public debt, net foreign assets, default episodes, and country risk for the thirty largest oil producing emerging economies in 2010.[4]

The data on oil reserves, oil production, oil net exports (thousands of barrels per day), and oil prices (Brent crude oil, USD per barrel) is from the US Energy Information Administration (EIA). For reserves, we used proved reserves.[5] For oil prices we use the real price by deflating the Brent spot price FOB with the US CPI index for all urban consumers all items

[4]Not all variables are available for all 30 countries. Appendix A documents the details by country regarding data availability.

[5]Reserves are difficult to measure given uncertainties about the quantity and quality of oil in the ground. Available measures include ultimately recoverable resources, proved, probable and possible reserves, and oil in place.

in US City average, seasonally adjusted (1982-1984=100).

As an indicator of country risk, we use the Institutional Investor's Index for Country Credit Ratings (III from now on). The III is an index of country risk published biannually in the March and September issues of the *Institutional Investor*. These credit ratings are based on information gathered from the Institutional Investor's Country Credit Survey, which reflects information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents grade each country on a scale of zero to 100, with 100 representing the smallest probability of default, and their responses are weighted according to their institutions' global exposure. The III is an indicator intended to capture a collection of risks related to investing in a particular country, including political risk, exchange rate risk, economic risk, and sovereign risk. We have biannual III data for the 1979-2016 period. The literature on sovereign risk typically uses spreads on sovereign debt measured with the Emerging Markets Bond Index (EMBI) as the measure of country risk. EMBI spreads, however, are only available since 1994 and for a small number of countries, which imposes limitations on the scope of the empirical analysis that we can conduct. For this reason we use the III. In Appendix B we show that the III is negatively correlated with EMBI spreads (meaning that risk moves in the same direction), and positively correlated with Moody's, and Fitch credit ratings.

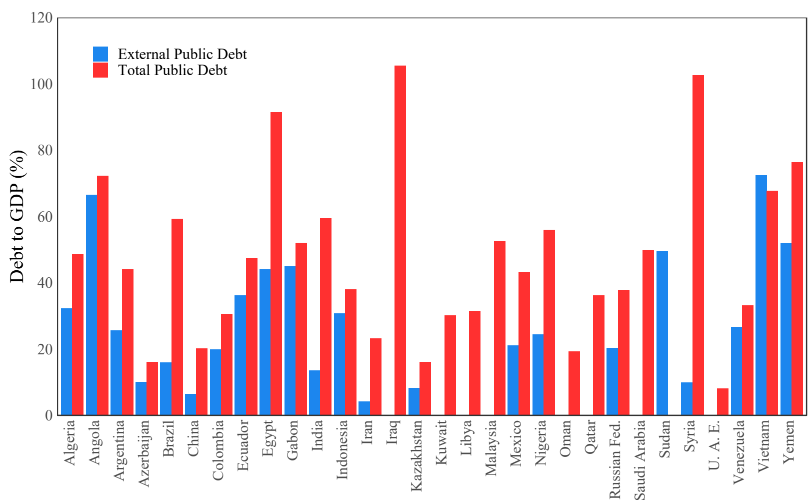
Total public external debt is from the World Bank Global Development Finance database (GDF), and net foreign assets from the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane & Milesi-Ferretti (2007). Default data is from Borensztein & Panizza (2009) for the 1979-2004 period and from Reinhart & Rogoff (2010) for the 2005-2014 period. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue (a restructuring).

### 2.2 Stylized Facts

The data we collected yields the following five key observations:

***1. Oil exporters have high external sovereign debt ratios and many have defaulted.*** Figure 2 shows the average external public debt to GDP ratio (in blue) and total public debt to GDP ratio (in red) for the thirty countries for which data is available in our sample. The lowest average external debt ratio is around 4% (Iran) and the largest is around 72% (Vietnam). Across all countries, the average external public debt to GDP ratio is roughly 29%. Figure 3 shows the number of default episodes—which ranges between zero and 5—for our full set of countries. [6]

Figure 2: Average External Public Debt of Net Oil Exporters (1971-2015)




***2. Higher external and total public debt are associated with higher country risk.*** Table 1 shows the unconditional correlation between the III and oil reserves, oil prices, external public debt to GDP, and total public debt to GDP for all the countries in our sample. Specifically, both external and total public debt are negatively correlated with the III, implying that they are positively correlated with sovereign risk (see columns IV and V of Table 1). This relationship is statistically significant in nearly all cases.

***3. Country risk decreases with oil prices but it has an ambiguous unconditional relationship with oil reserves.*** Oil prices and the III are positively correlated,[7] (see column (III) of

[6]Note that in Figure 3 a value of zero means that the country has not defaulted, it is not a lack of data.
[7]with the exception of Yemen, which has a non-significant negative correlation.

Table 1: Unconditional Correlations with the III

| (I) Country | (II) Oil Reserves | (III) Real Oil Prices | (IV) External Public Debt to GDP | (V) Total Public Debt to GDP |
|---|---|---|---|---|
| Algeria | 0.4109** | 0.7961*** | -0.7946*** | -0.7646*** |
| | (0.1564) | (0.1038) | (0.1041) | (0.1105) |
| Angola | 0.8148*** | 0.7940*** | -0.6758*** | -0.6862*** |
| | (0.0994) | (0.1043) | (0.1571) | (0.1669) |
| Argentina | 0.0409 | 0.3504** | -0.5593*** | -0.6367*** |
| | (0.1714) | (0.1606) | (0.1422) | (0.1342) |
| Azerbaijan | 0.5822** | 0.8102*** | -0.6679*** | -0.4679* |
| | (0.2347) | (0.1692) | (0.2148) | (0.2551) |
| Brazil | 0.8063*** | 0.8385*** | -0.7832*** | -0.0891 |
| | (0.1015) | (0.0934) | (0.1066) | (0.1708) |
| China | -0.4904*** | 0.8491*** | -0.8145*** | 0.6176*** |
| | (0.1495) | (0.0906) | (0.101) | (0.1436) |
| Colombia | -0.1011 | 0.8160*** | -0.7121*** | -0.1333 |
| | (0.1706) | (0.0991) | (0.1204) | (0.1725) |
| Ecuador | -0.0193 | 0.5475*** | -0.5580*** | -0.5799*** |
| | (0.1715) | (0.1435) | (0.1423) | (0.1397) |
| Egypt | -0.4412*** | 0.3151* | -0.6960*** | -0.5926*** |
| | (0.1539) | (0.1628) | (0.1231) | (0.1402) |
| Gabon | -0.3439** | 0.6552*** | -0.7171*** | -0.7340*** |
| | (0.161) | (0.1296) | (0.1195) | (0.1165) |
| India | 0.1002 | 0.7671*** | -0.8526*** | 0.1099 |
| | (0.1706) | (0.11) | (0.0896) | (0.1757) |
| Indonesia | 0.1843 | 0.4997*** | -0.5223*** | -0.8698*** |
| | (0.1686) | (0.1485) | (0.1462) | (0.0846) |
| Iran | 0.6619*** | 0.0096 | 0.1193 | -0.6336*** |
| | (0.1286) | (0.1715) | (0.1703) | (0.1327) |
| Iraq | -0.5051*** | 0.6645*** | | -0.8239*** |
| | (0.148) | (0.1282) | | (0.1792) |
| Kazakhstan | 0.8242*** | 0.8552*** | -0.5290*** | -0.7276*** |
| | (0.1373) | (0.1105) | (0.1809) | (0.1497) |
| Kuwait | 0.0090 | 0.8031*** | | -0.8869*** |
| | (0.1715) | (0.1022) | | (0.0804) |
| Libya | 0.4365*** | 0.7220*** | | -0.7439*** |
| | (0.1543) | (0.1187) | | (0.1146) |
| Malaysia | 0.2579 | 0.7556*** | | -0.2100 |
| | (0.1657) | (0.1123) | | (0.1702) |
| Mexico | -0.8342*** | 0.7188*** | -0.7311*** | -0.5777*** |
| | (0.0946) | (0.1192) | (0.117) | (0.1421) |
| Nigeria | 0.3966** | 0.8486*** | -0.7694*** | -0.6637*** |
| | (0.1574) | (0.0907) | (0.1096) | (0.1283) |
| Oman | -0.3821** | 0.8486*** | | -0.8124*** |
| | (0.1585) | (0.0907) | | (0.1) |
| Qatar | 0.9141*** | 0.7771*** | | -0.3789* |
| | (0.0696) | (0.1079) | | (0.1889) |
| Russian Federation | 0.7043*** | 0.7162*** | -0.7616*** | -0.8646*** |
| | (0.1722) | (0.1197) | (0.1382) | (0.1071) |
| Saudi Arabia | -0.2988* | 0.8583*** | | -0.9189*** |
| | (0.1637) | (0.088) | | (0.0823) |
| Sudan | 0.5307*** | 0.6223*** | -0.6420*** | -0.7663*** |
| | (0.1498) | (0.1342) | (0.1315) | (0.137) |
| Syria | 0.3769** | 0.2753 | -0.9645*** | -0.7378*** |
| | (0.1589) | (0.1649) | (0.1078) | (0.1253) |
| United Arab Emirates | 0.3827** | 0.6602*** | | 0.5593*** |
| | (0.1584) | (0.1288) | | (0.1422) |
| Venezuela | -0.4032** | 0.4162** | -0.4311*** | -0.6867*** |
| | (0.1569) | (0.1559) | (0.1571) | (0.1374) |
| Vietnam | 0.3953* | 0.8265*** | -0.8309*** | -0.7316*** |
| | (0.1958) | (0.12) | (0.1186) | (0.1488) |
| Yemen | 0.4599 | -0.2085 | 0.5063* | -0.6523** |
| | (0.2677) | (0.2949) | (0.26) | (0.2285) |

Standard errors in parenthesis
*** p<0.01, ** p<0.05, * p<0.1

Figure 3: Net Oil Exporters Number of Default Episodes (1979-2014)

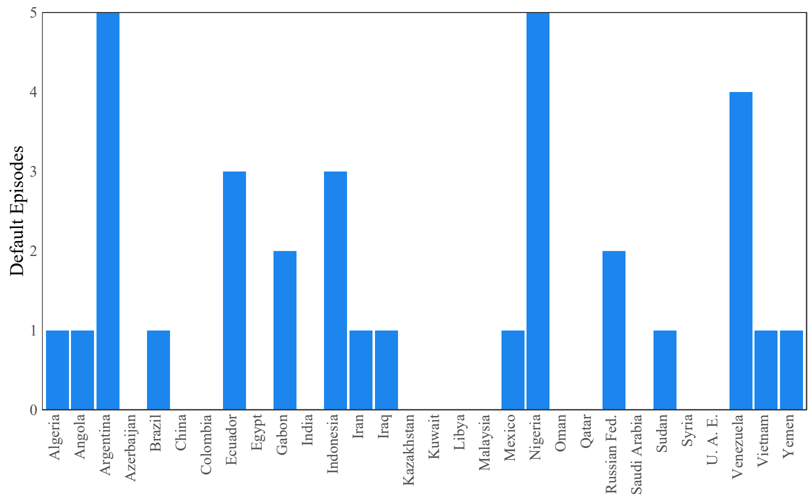



Table 1), while reserves and the III are positively correlated in some countries and negatively correlated in others (see column (II) of Table 1).

***4. Higher oil production is associated with lower country risk but higher reserves are associated with higher country risk.*** We established these empirical results by examining the data in two ways. First, by estimating unconditional between-means panel regressions of the III on oil production and on the ratio of oil reserves to production. These are simple OLS regressions estimated with the country full-sample averages of each variable.[8]

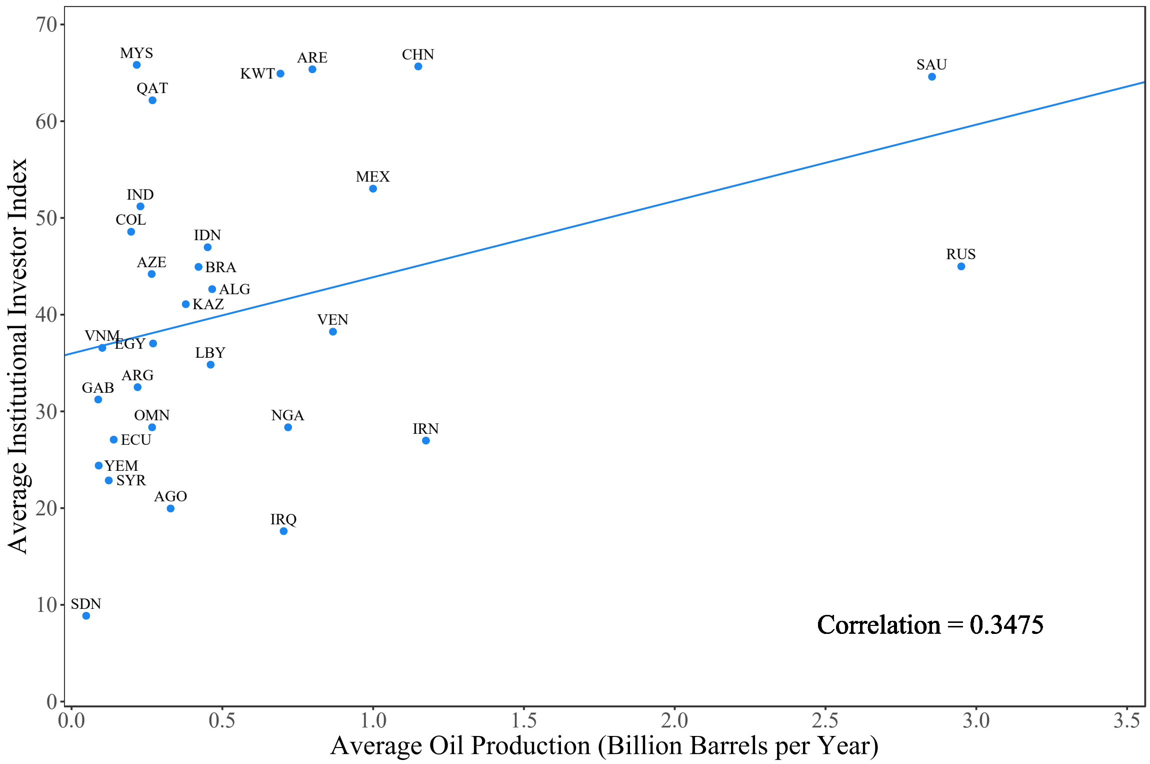
Figure 4 shows results for regressing the III on average oil production indicating that, in the long-run, countries that maintained higher mean oil production had a slightly higher average credit rating, with a regression coefficient of 0.28.[9] On the other hand, the regression of the III on the oil reserves to production ratio, shown in Figure 5, indicates that the two variables are not correlated as the regression coefficient is -0.02. However, non of the two regression coefficients are statistically significant.

These between-means regressions have the limitation that they do not separate short-

[8]Reserves to production (i.e. oil extraction) represents the number of years that it would take a country to deplete its reserves assuming that they keep extracting at the same rate and there are no new discoveries.

[9]SAU and RUS appear to be outliers in Figure 4, if we remove them and run the same regression we get a correlation coefficient of 0.31, but the regression coefficient goes up

Figure 4: In between effects regression of the Institutional Investor Index (Y-Axis) on average oil production (X-Axis): 1980-2016.

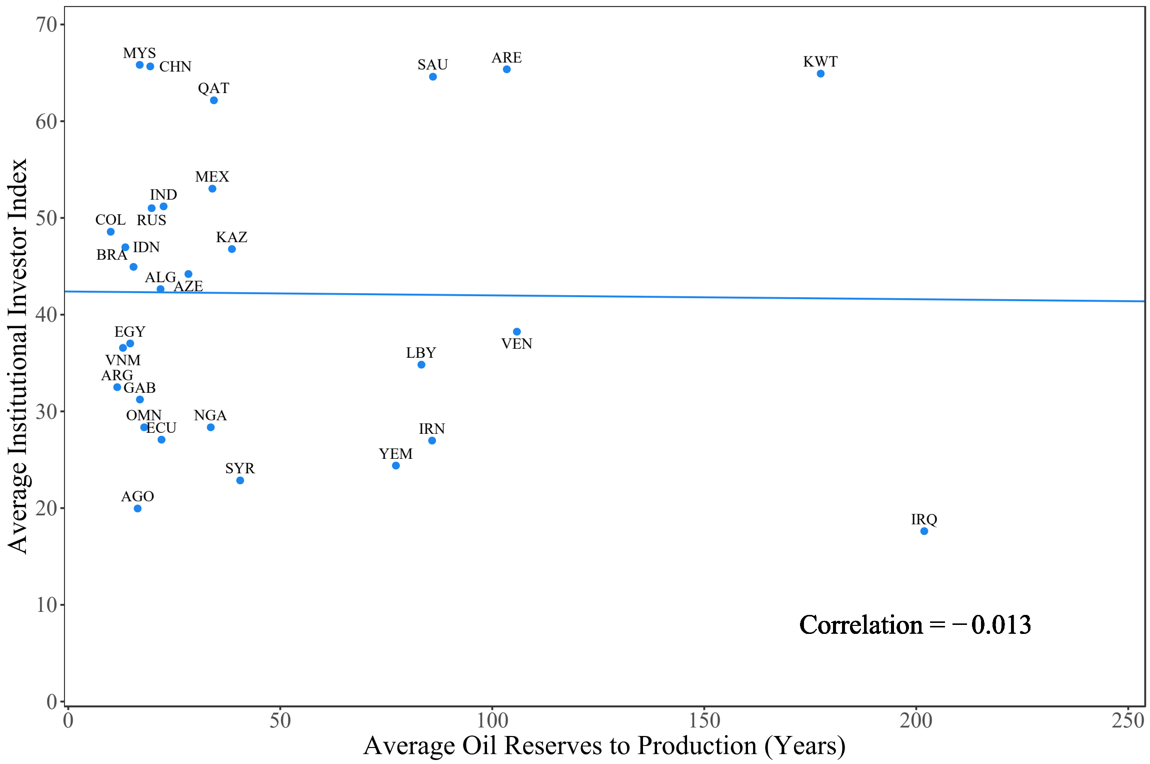



from long-run effects in the dynamic relationship between country risk and oil variables and do not condition for any relevant control variables. Hence, the second approach we followed to study the comovement between country risk and oil variables is based on full dynamic panel regressions.

Notice that our analysis has two dimensions. We want to study the effect of oil production (the flow) versus the effect of oil reserves (the stock) both in the short and the long-run. Thus far, these unconditional correlations point towards two mechanisms. First, extracting more oil (production) increases a country's ability to repay its debt, decreasing country risk. Second, owning a larger stock of oil (reserves) seems to be positively correlated with country risk, and this goes in line with the idea that if a country has a larger stock of a real asset, then financial autarky becomes a more attractive option.

In order to study more formally the presence of these two mechanisms in the data, and establish conditional correlations, we run a dynamic fixed effects estimation of long-run, short-

Figure 5: In between effects regression of the Institutional Investor Index (Y-Axis) on average oil Reserves to production (X-Axis): 1980-2016.




Note: Sudan is removed from the calculations because it is an outlie.

run and convergence coefficients. This allows us to put all the previous results together and be able to establish statistical significance of the relevant variables and timing. The results are shown in Table 2.[10]

We estimate the dynamic panel regressions for three different model specifications. In Model (1) we regress the III on oil production, real non-oil GDP in local currency units, oil reserves, external public debt to GDP, oil discoveries, and a default dummy. In Model (2) we control for net foreign assets and exclude the default dummy, and in Model (3) we control for everything. In the three model specifications we control for country fixed effects (to take

[10]Due to data limitations, Azerbaijan, Kazakhstan, Kuwait, Iraq, Libya, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen are dropped from the dynamic fixed effects regressions. Consequently, the estimation is performed taking into account 512, 509 and 509 observations in Model 1, 2, and 3 respectively.

care of particular political situations), as well as for time fixed effects (to control for oil-price effects). All coefficients can be interpreted as elasticities with the exception of the coefficient on oil discoveries. We ran the regressions for both a balanced and an unbalanced panel. Table 2 shows the results for the unbalanced panel, but the results are robust in the balanced case.

Table 2: Dynamic Fixed Effects Regression Results for Institutional Investor Index

| | Δ Inst. Investor Index | | |
|---|---|---|---|
| | Model (1) | Model (2) | Model (3) |
| **Convergence coefficient** | | | |
| Inst. Investor Index (-1) | -0.175*** | -0.156*** | -0.183*** |
| | (0.019) | (0.020) | (0.020) |
| **Short-run coefficients** | | | |
| Δ Oil Production | 0.052** | 0.047** | 0.055** |
| | (0.021) | (0.022) | (0.022) |
| Δ Non-Oil GDP | 0.199*** | 0.231*** | 0.198*** |
| | (0.058) | (0.059) | (0.057) |
| Δ Oil Reserves | 0.006 | 0.014 | 0.010 |
| | (0.020) | (0.020) | (0.020) |
| Δ Ext. pub. debt to GDP | -0.104*** | -0.094* | -0.107** |
| | (0.038) | (0.052) | (0.051) |
| Δ Oil Discoveries | -0.003 | -0.003 | -0.003 |
| | (0.003) | (0.004) | (0.003) |
| Δ NFA | | -0.040 | -0.046 |
| | | (0.035) | (0.034) |
| **Long-run coefficients** | | | |
| Oil Production | 0.048 | 0.048 | 0.038 |
| | (0.041) | (0.049) | (0.041) |
| Non-oil GDP | 0.095 | -0.027 | 0.101 |
| | (0.106) | (0.120) | (0.100) |
| Oil Reserves | -0.162*** | -0.141** | -0.141*** |
| | (0.051) | (0.060) | (0.050) |
| Ext. pub. debt to GDP | -0.810*** | -1.226*** | -1.001*** |
| | (0.140) | (0.219) | (0.178) |
| Default | -0.369*** | | -0.379*** |
| | (0.072) | | (0.068) |
| Oil Discoveries | 0.045 | 0.048 | 0.039 |
| | (0.028) | (0.033) | (0.027) |
| NFA | | -0.003 | -0.119 |
| | | (0.141) | (0.116) |
| Constant | 0.245 | 0.767 | 0.219 |
| | (0.542) | (0.546) | (0.537) |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

The convergence coefficient measures the speed at which the III converges to its long-run average. As such, in each model the convergence coefficient has the expected sign and is statistically significant at the $1\%$ level. Convergence in the III runs at an annual rate in between $0.156\%$ and $0.183\%$, which means that each year the III covers about $0.17\%$ (depending on the model) of its distance to the "steady state." It should also be noted that convergence is slightly slower in Model (2), where the net foreign assets-to-GDP ratio is included and default is excluded.

If we focus on the short-run coefficients, we observe that an increase of $1\%$ in oil production decreases country risk around $0.05\%$ at the $5\%$ significance level. An increase in non-oil GDP decreases country risk, and this result is significant to a $1\%$ level. In the short run, a positive change in oil reserves (which can happen if extraction is lower than discoveries of oil in a given period), decreases country risk, but the coefficient is not significant. As is expected, increases in external public debt increase country risk, and this result is statistically significant at $1\%$, $10\%$ and $5\%$ significance levels respectively. Finally, a positive *change* in net-foreign assets increases country risk but the coefficient is not significant.

When looking at the long-run coefficients, as shown in Pesaran et al. (1999), the usual interpretation—when series are in logs—is that of an elasticity. Then, the long-run oil production elasticity is $0.05$ in the first and second model, and $0.04$ in the third, which means that when oil GDP increases by $1\%$, the III is between $0.04\%$ and $0.05\%$ higher in the long-run, however the coefficients are not significant. With respect to non-oil GDP, long-run elasticities are positive in two cases, but none is statistically significant.

Moreover, a significant negative relationship between oil reserves and the III was found. A rise in oil reserves of $1\%$ worsens our measure of country risk in the long term by about $0.15\%$. Thus, an oil exporting economy is perceived as more risky in the future when it boosts its reserves today. This elasticity is statistically different from zero at a $5\%$ level for the second model, and at a $1\%$ level for the first model and third model where we control for net-foreign assets, default, and oil discoveries.

As expected, in the long-run, external public debt still has a negative effect on country risk and is statistically significant to a $1\%$ level for the three models. Similar to the short run, in the long run the *level* of net-foreign assets increases country risk, but it is not statistically significant. Finally, as expected, being in default increases country risk. When a country is in default, the III drops about $37\%$. This last result is statistically significant to a $1\%$ level.

As for oil discoveries, an increase in oil discoveries decreases country risk but the effect is not statistically significant in all three models.

These results support the two mechanisms that we believe are behind the unconditional correlations presented above. Oil production decreases country risk by increasing a country's ability to repay in the short run, but greater oil reserves increase country risk by making autarky more attractive in the long run.

This results suggest that there is a trade-of between the financial asset (public debt) and the real asset (oil). If I have a larger stock of oil, then I can increase production to smooth consumption, rather than borrowing in financial markets. This trade off should rely on the relative yield of the two assets, or in other words, on the price of oil and the price of sovereign debt.

***5. Cycles in oil prices are associated with business cycles, and differ between countries that have defaulted and those that have not.*** We illustrate this relationship in Tables 3 to 5 and in Figure 6. Tables 3, 4, and 5 show the mean and the standard deviation of the main variables of interest, as well as their correlation with GDP and the real price of oil for the thirty countries in our sample, for just the set of countries that have defaulted, and for just the set of countries that have not defaulted, respectively. Detrended variables have a mean of zero.

Table 3 highlights the fact that debt is negatively correlated with the oil price, showing that at high oil prices, this set of countries acquires less debt than at low oil prices. Reinforcing once more the idea that agents use either their financial or their real asset to smooth out consumption depending on relative yields. This is true for all thirty countries irrespective of whether they are defaulters or not as well as the fact that oil prices are positively correlated with the III which also shows that higher oil prices are associated with less risk (see Tables 4 and 5). Non-defaulters have a positive correlation with the TB, which shows that at higher prices exports of oil increase. However, for the set of defaulters this correlation is close to zero.

Two facts that are different about this set of emerging countries, is that consumption is less volatile than output, and the trade balance is pro-cyclical as opposed to counter-cyclical (see Neumeyer & Perri (2005); Restrepo-Echavarria (2014)). This is due to the presence of the Arab countries.

Figure 6 shows the relationship between oil price upswings and downswings and some macro variables. To construct this figure we divided the panel data in two. In one group we have all years where oil prices were increasing (oil-price upswings), and in a second group we have all years where oil prices were decreasing (oil-price downswings). Table D.1 (see Appendix D) shows how each year corresponds to a downswing or an upswing. We then averaged the different macroeconomic variables over the upswings and downswings and Figure 6 shows the results for the relationship between the upswings, downswings, and different macro variables.

Oil price upswings are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price downswings are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and higher number of default events.

Figure 6: Oil Price Swings and Macro Performance

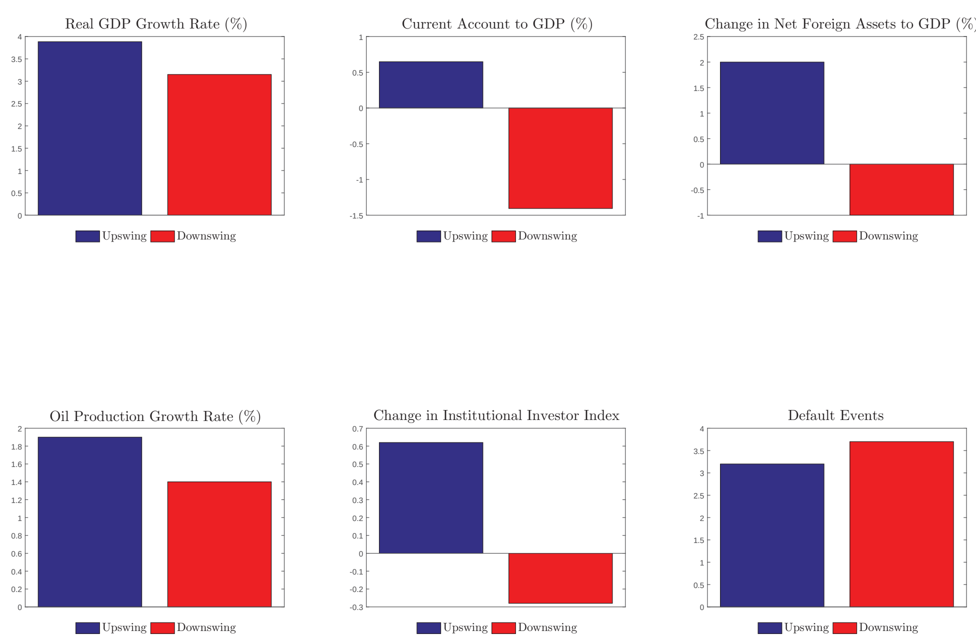


Table 3: Oil Prices and Business Cycle Moments 30 Countries

| | Mean | Std.dev. | Corr(i,GDP) | Corr(i,Oil Price) | Acorr |
|---|---|---|---|---|---|
| Oil Price | | 0.182 | 0.11 | 1.00 | 0.85 |
| Non-Oil GDP | | 0.092 | 0.63 | -0.04 | 0.38 |
| GDP | | 0.069 | 1.00 | 0.11 | 0.52 |
| Oil Production | | 0.122 | 0.62 | 0.04 | 0.50 |
| Consumption | | 0.049 | 0.52 | 0.12 | 0.52 |
| TB/GDP | 0.073 | 0.089 | 0.11 | 0.19 | 0.62 |
| III | 0.475 | 0.115 | 0.21 | 0.69 | 0.86 |
| Debt/GDP | 0.224 | 0.144 | -0.28 | -0.61 | 0.83 |
| Gross Oil Output/GDP | 0.282 | 0.145 | 0.11 | 0.46 | 0.64 |

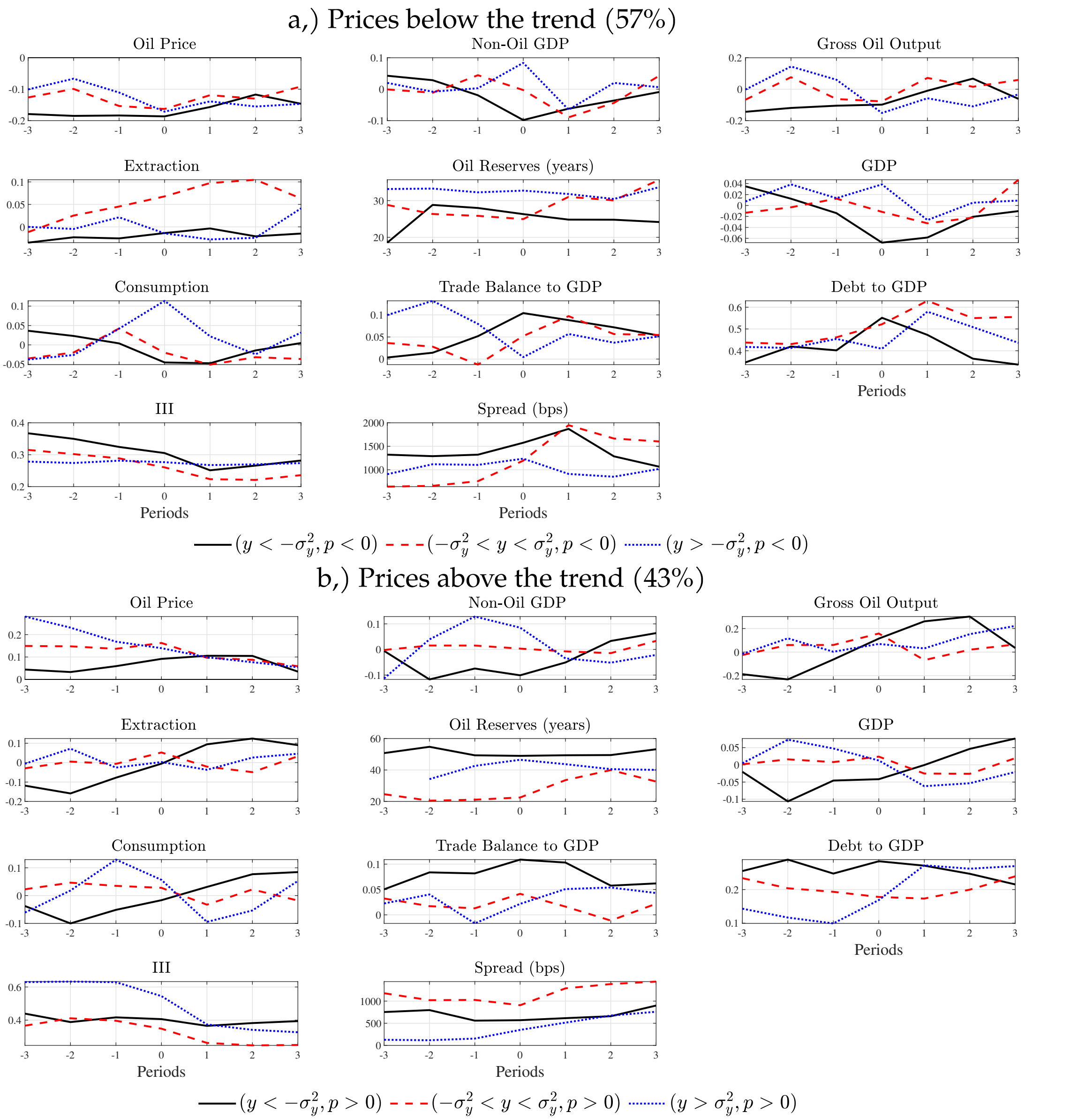
Table 4: Oil Prices and Business Cycle Moments Defaulters

| | Mean | Std.dev. | Corr(i,GDP) | Corr(i,Oil Price) | Acorr |
|---|---|---|---|---|---|
| Oil Price | | 0.182 | 0.14 | 1.00 | 0.85 |
| Non-Oil GDP | | 0.072 | 0.72 | 0.00 | 0.42 |
| GDP | | 0.064 | 1.00 | 0.14 | 0.50 |
| Oil Production | | 0.114 | 0.62 | 0.06 | 0.52 |
| Consumption | | 0.052 | 0.72 | 0.15 | 0.56 |
| TB/GDP | 0.048 | 0.063 | 0.03 | 0.03 | 0.54 |
| III | 0.376 | 0.137 | 0.27 | 0.62 | 0.87 |
| Debt/GDP | 0.249 | 0.161 | -0.32 | -0.61 | 0.82 |
| Gross Oil Output/GDP | 0.237 | 0.157 | 0.07 | 0.37 | 0.67 |

Table 5: Oil Prices and Business Cycle Moments Non-Defaulters

| | Mean | Std.dev. | Corr(i,GDP) | Corr(i,Oil Price) | Acorr |
|---|---|---|---|---|---|
| Oil Price | | 0.182 | 0.08 | 1.00 | 0.85 |
| Non-Oil GDP | | 0.117 | 0.52 | -0.10 | 0.34 |
| GDP | | 0.074 | 1.00 | 0.08 | 0.54 |
| Oil Production | | 0.133 | 0.62 | 0.01 | 0.49 |
| Consumption | | 0.046 | 0.26 | 0.08 | 0.47 |
| TB/GDP | 0.102 | 0.120 | 0.19 | 0.37 | 0.72 |
| III | 0.588 | 0.089 | 0.13 | 0.77 | 0.86 |
| Debt/GDP | 0.139 | 0.081 | -0.16 | -0.60 | 0.88 |
| Gross Oil Output/GDP | 0.335 | 0.132 | 0.16 | 0.56 | 0.60 |

Figure 7: Three different types of default in the data




***6. Different types of defaults.*** In the data we observe that defaults don't always occur under the same circumstances, so we think its important to illustrate the different types of default rather than just showing the combined event windows for all cases, as is usually the case. Figure 7 has two panels that jointly illustrate six different types of defaults in the data. Panel (a) collects those defaults that happen when oil prices are below trend, and panel (b) those defaults that occur when prices are above trend, and both panels contain three lines corresponding to the case when non-oil output is one standard deviation or more below trend (black line), the case when non-oil output is within one standard deviation of its trend (dashed red line), and the case when non-oil output is one or more standard deviations above trend (dotted blue line). We see that the majority of default episodes (at $57\%$), occur when prices are below their trend, and $43\%$ occur when prices are above trend.

Panel (a) shows that in terms of total GDP, there are two types of defaults. Those that occur when GDP is falling, either because it already was below trend and it keeps falling (black line) or because it was slightly above trend and it falls below trend (dashed red line), and those that occur when GDP is already above trend and rises further (dotted blue line). We characterize the former as defaults that occur in bad times, and the latter as defaults that occur in good times.

Defaults that occur in good times—with the price of oil below trend and decreasing and non-oil output increasing above trend—are fundamentally different from those that happen in bad times. Because non-oil GDP is increasing above trend (overcompensating the drop in oil prices), GDP increases while oil extraction goes down because it is not worth extracting oil at those prices plus its not necessary. The increase in GDP is enough to generate an increase in consumption while reducing debt and the trade balance. However, in those defaults that happen in bad times, we can see that both extraction (the real asset) and debt (the financial asset) go up to compensate the drop in non-oil GDP, and smooth out consumption as much as possible, albeit not preventing it from dropping.

As mentioned above, Panel (b) illustrates those that occur when oil prices are above trend. In the same way as in panel (a), we can see that there are defaults that happen in good times (dashed red line), and defaults that happen in bad times (solid black line and dotted blue line). If we aggregate these two types of defaults (good versus bad times) for panel (a) and panel (b) of Figure 7, we find that half of the default episodes happen in good times and the other half in bad times. This result means that in the countries in our sample the probability of a default happening in good times is higher than that found by Tomz & Wright (2007), who find that that probability is $30\%$. However our definition of good and bad times is not the same.

Facts 1 and 2 are important because they highlight the relevance of studying the role of oil in business cycles and the sovereign debt and default dynamics of oil exporting emerging economies (and of commodity producers more broadly), but they are fairly well-known facts.

However, facts 3, 4, and 5 are particularly important new facts. First, points 3 and 4 indicate that oil has two opposing effects on country risk; higher oil production and/or prices reduce country risk (maybe by increasing a country's ability to repay), but higher oil reserves do not reduce country risk (maybe because they indicate a higher default option value). Point 5, shows that not all defaults are the same, they happen under different circumstances and have different implications for macroeconomic variables. The task for the remainder of the paper is to determine whether a model of sovereign default that endogenizes oil extraction and reserve accumulation can be consistent with all of these empirical regularities.

## 3 A Model of Sovereign Default and Oil Extraction

The model we propose is in the class of those based on the work of Eaton & Gersovitz (1981), in which a benevolent social planner cannot commit to repay external debt and chooses optimally whether to default or not. The key difference is that we introduce optimal oil extraction and reserves decisions. The planner owns the oil industry, and thus chooses oil extraction and reserves.[11] This is a nontrivial modification because it implies that the planner now has two vehicles for reallocating resources intertemporally (debt and oil reserves) and can affect the option value of default by altering oil reserves. In addition, the planner's income and repayment capacity are exposed to the risk of oil-price shocks.

### 3.1 Model structure

There are two types of goods in the model, oil and a tradable, non-storable consumption good. The price of oil relative to the consumption good, $p$, is stochastic and determined in world markets. Hence, the sovereign is a price-taker in the world oil market. The economy has an exogenous stochastic endowment of the tradable good (non-oil GDP), $y$, which has an exogenous world-determined price set to 1 without loss of generality. Oil prices and non-oil GDP follow a joint first-order, stationary Markov process with known realization vectors and a transition probability matrix denoted by $\pi(p', y'|p, y)$.

[11] This assumption is in line with the dominant role of state-owned enterprises in commodity extraction and/or exports in many emerging and developing economies.

Oil is extracted at a cost denominated in units of the consumption good. The cost of extracting $x$ units of oil out of an existing stock of oil reserves $s$ is determined by the extraction cost function $e\left(x,s\right)$, so that oil GDP is $y^O \equiv px - e(x,s)$. The extraction cost function has these properties: $e_s < 0$, $e_x > 0$ and $e_s\left(0,s\right) = 0$. Its functional form is as follows:

$$e(x,s) = \psi\left(\frac{x}{s}\right)^{\gamma} x. \tag{1}$$

Hence, the per-unit extraction cost ($\psi\left(\frac{x}{s}\right)^{\gamma}$) is homogeneous of degree zero in extraction and reserves.

Reserves follow the law of motion $s' = s - x + \kappa$, where $\kappa$ denotes a constant amount of oil discoveries each period and $s'$ denotes reserves carried over to the next period. Extraction cannot be negative ($x \geq 0$) and cannot exceed the sum of reserves plus discoveries ($x \leq s + \kappa$). Since oil is a form of capital with an endogenous return, it has an asset valuation that we label the "asset price of oil" defined as $q^O \equiv p - e_x(x,s) + \Delta\tilde{\psi}$, where $\Delta\tilde{\psi} \equiv [\psi^l - \psi^h]/u'(c)$ and $\psi^l$ and $\psi^h$ are the multipliers on the lower and upper bounds of $x$, respectively.[12]

The world credit market is the same as in standard Eaton-Gersovitz (EG) models. A benevolent government maximizes private-sector utility, defined by a standard time-separable expected utility function with constant-relative-risk-aversion period utility $u(c) = \frac{c^{1-\mu}}{1-\mu}$ and subjective discount factor $\beta$. The sovereign sells one-period, non-state contingent discount bonds denominated in units of the consumption good to risk-neutral foreign investors. The outstanding bond position is denoted $b$ and newly issued bonds are denoted $b'$ (the sovereign is indebted when $b < 0$). The set of feasible bond positions is given by a discrete grid defined over the interval $B = [b_{min}, b_{max}]$where $b_{min} \leq b_{max} = 0$. The sovereign cannot commit to repay the debt. If it defaults, it does not repay $b$ in the current period and is excluded from the credit market in the same period, so no $b'$ can be issued. Next period, the sovereign can re-enter the credit market with probability $\lambda$. We also assume that the country can still participate in the oil market during the exclusion period. Hence, the sovereign can still export oil when it defaults. This is important because it implies that the sovereign's plans for the accumulation of oil reserves affect the value of default, since those reserves can be extracted and exported to generate income while access to credit markets remains closed. In contrast,

[12] Appendix F shows that, taking as given a bond pricing function, $q^O$ equals the expected present discounted value (discounted with the sovereign's stochastic discount factor) of the income stream composed of oil "dividends", $d^O \equiv -e_s(t) + \psi^h_{t+1}/u'(c_t)$, and the marginal revenue resulting from the effect of accumulating higher oil reserves on the price of debt.

in EG models the value of default is typically exogenous to the government's decisions.

The timing of decisions within a period is as follows: At the beginning of the period, $s$ and $b$ are known. The shocks $p$ and $y$ are realized. The sovereign then decides whether to repay or default on $b$ by choosing the option that yields the highest value, as explained below. If the sovereign defaults, it makes oil extraction and reserves decisions, since the country is excluded from the world bond market but not from the oil market, and pays extraction costs. If the sovereign repays, it sells new bonds $b'$ to foreign investors at the price $q$, makes extraction and reserves decisions, and pays extraction costs. The resources generated from debt and profits from oil exports are then transferred to households and used for consumption.

The planner's payoff at beginning of the period is:

$$V(b, s, y, p) = \max \left\{ v^{nd}(b, s, y, p), v^{d}(s, y, p) \right\},$$

where $v^{nd}(b, s, y, p)$ is the value of no-default and $v^{d}(s, y, p)$ is the value of default.

The value of no-default is characterized by the following constrained maximization problem:

$$v^{nd}(b, s, y, p) = \max_{\{c,x,b',s'\}} \left\{ u(c) + \beta E \left[ V\left(b', s', y', p'\right) \right] \right\} \tag{2}$$

subject to the following constraints:

$$c = y - A + px - e\left(x, s\right) + b - q\left(b', s', y, p\right) b', \tag{3}$$

$$s' = s - x + \kappa, \tag{4}$$

$$0 \leq x \leq s + \kappa. \tag{5}$$

Constraint (3) is the resource constraint, (4) is the law of motion of oil reserves, and (5) represents the feasibility constraints on extraction. In the resource constraint, $q\left(b', s', y, p\right)$ is the pricing function for the risky sovereign bond, which will depend in equilibrium on the choices of bonds and reserves and the realizations of $(p, y)$, and $A$ represents autonomous (exogenous) spending allocated to investment expenditures so that the consumption-GDP ratio can be calibrated later to match the data (consumption will include private and public consumption). Note that we are assuming that extraction costs are factor payments abroad.[13]

[13] This assumption can be relaxed by assuming that a fraction $\phi$ of extraction costs are domestic factor income. In which case $e(x, s)$ is replaced with $(1 - \phi)e(x, s)$ in the resource constraint.

The value of default is characterized by the following constrained optimization problem:

$$v^d(s,y,p) = \max_{\{c,x,s'\}} \left\{ u(c) + \beta\left(1-\lambda\right) E v^d\left(s',y',p'\right) + \beta\lambda E V\left(0,s',y',p'\right) \right\} \tag{6}$$

subject to the same law of motion of reserves and feasibility constraint as in the no-default case and the following resource constraint:

$$c = y - A + h(p)x - e\left(x,s\right). \tag{7}$$

In the right-hand-side of the value of default (6), the sovereign re-enters credit markets with probability $\lambda$ and a clean slate of debt ($b' = 0$), and it retains its oil reserves $s'$. It remains in default with probability $(1-\lambda)$ but again it retains its oil reserves $s'$. The resource constraint (7) includes a piece-wise default cost akin to the one proposed by Arellano (2008) for income but in terms of the price of oil: $h(p) = \hat{p}$ if $p > \hat{p}$ and $h(p) = p$ if $p \leq \hat{p}$. Intuitively, this is similar to a foreign ad-valorem tariff on the country's oil exports that rises with $p$ above the threshold $\hat{p}$. This trade penalty is in line with the empirical observation that international trade is negatively affected by sovereign default. Alternatively, we can focus on the implied income default cost that $h(p)$ induces in terms of oil output or aggregate GDP. Both are affected not only by the exogenous adjustment in $p$ but by the endogenous response of oil production (and hence of total GDP) induced by that adjustment. Hence, unlike in most EG models, this model's default cost in terms of income includes an endogenous component. We examine this issue in more detail in Sections 4.1 and 4.4.

For a given $(b,s)$, default is optimal for the pairs $\{y,p\}$ for which $v^d(s,y,p) \geq v^{nd}(b,s,y,p)$. Hence, the default set is given by:

$$D\left(b,s\right) = \left\{ \{y,p\} : v^d(s,y,p) \geq v^{nd}(b,s,y,p) \right\}. \tag{8}$$

The default decision rule associated with this default set is given by the function $d(b,s,y,p)$, which takes the value of 1 for $(y,p) \in D\left(b,s\right)$ and 0 otherwise (i.e. it equals 1 if the government defaults).

The probability of default one-period ahead conditional on current-period information, $P^d(b',s',y,p)$, can then be induced from the default decision rule and the Markov process of the shocks as follows:

$$P^d\left(b',s',y,p\right) = \sum_{y'}\sum_{p'} d(b',s',y',p')\pi(y',p'|y,p). \tag{9}$$

Since foreign investors are risk neutral, sovereign bond prices are determined by the standard no-arbitrage condition:

$$q\left(b', s', y, p\right) = q^* \left(1 - P^d\left(b', s', y, p\right)\right),$$

where $q^*$ is the price of a risk-free bond such that $q^* \equiv 1/R^*$ where $R^*$ is the world's risk-free gross real interest rate that represents the opportunity cost of funds for foreign investors.

### 3.2 Model properties

Appendix G includes six propositions that show useful features of the asset price of oil and oil profits, demonstrate that some of the properties of the standard EG model hold in this setup, and characterize the effects of oil reserves and oil-price shocks. Relative to the standard model, obtaining analytic results is more difficult because of the endogeneity of the default payoff on the choice of oil reserves (whereas in most EG models the default payoff is exogenous). As we explain below, this is particularly the case for deriving results related to how reserves affect default risk, what contracts are feasible under repayment when default is possible, and how default incentives respond to $y$ and $p$ shocks.

The propositions rely on three conjectures: 1) Asset prices of oil are non-negative under repayment and default; 2) optimal consumption under repayment is nondecreasing in $s$; and 3) for $(y, p)$ pairs in the default set when this set is non-empty, the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits between repayment and default.

Since the propositions rely on these conjectures, and some impose parameter restrictions (i.i.d shocks, permanent exclusion after default, and no oil-price default cost or $\hat{p} = p$) and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions using the calibration specified in the next Section. They all hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 which holds in 98 percent of the corresponding evaluations (see Appendix G for details). We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 5 and 6.[14] Profits are strictly positive for all optimal decision rules of $s'$ under repayment and default. The trade balance conditions of Propositions 5 and 6

[14]We also checked whether the boundary conditions for $x$ (or $s'$) bind and found that they are never binding.

hold 97 and 100 percent of all model evaluations, respectively. Removing the trade balance conditions, the main results of those propositions, namely that default incentives strengthen at lower $y$ (Proposition 5) or lower $p$ (Proposition 6) also hold in 100 percent of the model evaluations. Thus, in our calibrated numerical solution, lower oil prices *always* strengthen default incentives and the sufficiency condition to prove it (i.e., the trade balance condition of Proposition 6) always holds. Lower $y$ *always* strengthens default incentives and the sufficiency condition to prove it (i.e., the trade balance condition of Proposition 5) holds in 97 percent of the evaluations.

**Proposition 1. The repayment payoff is non-decreasing in $b$ and default sets shrink as $b$ rises (i.e. grow as debt rises)**

*For all $b^1 \leq b^2$, $v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$. Moreover, if default is optimal for $b^2$ ($d(b^2, s, y, p) = 1$) for some states $(s, y, p)$ then default is optimal for $b^1$ for the same states $(s, y, p)$ (i.e. $D(b^2, s) \subseteq D(b^1, s)$ and $d(b^1, s, y, p) = 1$) .*

This is analogous to Proposition 1 in Arellano (2008). It implies that the country risk premium is non-decreasing in the amount of new debt issued ($q(\cdot)$ is non-decreasing in $b'$).

**Proposition 2. If asset prices of oil are positive, oil profits are increasing in $s$, for given $s'$, and decreasing in $s'$, for given $s$.**

*Given Conjecture 1, oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M^{nd}_s(\cdot), M^d_s(\cdot) > 0$, and decreasing in $s' \in [s+\kappa-s(p/\psi)^{(1/\gamma)}, s+\kappa]$, namely $M^{nd}_{s'}(\cdot), M^d_{s'}(\cdot) < 0$.*[15]

This proposition shows that, if the asset prices of oil are positive under repayment and default, the corresponding profits from oil extraction are higher if reserves carried over from the previous period are higher, for a given value of $s'$, and lower if new reserves are higher (i.e. extraction falls) for a given value of $s$. We show in Appendix F that positive asset prices of oil are equilibrium outcomes in three variants of the model without default risk (financial autarky and an exogenous bond pricing function set equal to $q^*$ or to a function with the same properties as that of a model with default). The result under financial autarky implies also that $q^{Od}(\cdot) > 0$ in the model with default and $\lambda = 0$.

**Proposition 3. The default and repayment payoffs are non-decreasing in s.**

*For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ and $v^d(s^2, y, p) \geq v^d(s^1, y, p)$.*

This result follows from Proposition 2, and demonstrates that one of the conditions needed

[15]The lower bound of $s'$ follows from assuming oil profits are non-negative. The upper bound is at the point where extraction is set to zero. See Appendix G for details.

for the default sets to shrink in $b$ in Proposition 1 (namely that the default and repayment payoffs are non-decreasing in $b$) also applies with respect to $s$. This is not sufficient, however, to yield the result that default sets shrink in $s$, as the next proposition shows.

**Proposition 4. Default sets shrink as $s$ rises (i.e. grow as reserves fall).**

*Assume $\hat{p} = p$ and $\lambda = 0$ for simplicity. For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, if default is optimal for $s^2$ ($d(b, s^2, y, p) = 1$) for some states $(b, y, p)$, then default is optimal for $s^1$ for the same states $(b, y, p)$ (i.e. $D(b, s^2) \subseteq D(b, s^1)$ and $d(b, s^1, y, p) = 1$).*

This proposition establishes sufficiency conditions under which the result about country risk with respect to the bond position established in Proposition 1 extends to oil reserves. It relies on the three conjectures and Propositions 2 and 3 and establishes that the country risk premium is non-decreasing in the choice of $s'$ (i.e., $q(\cdot)$ is non-decreasing in $s'$). This result does not follow just from analogy to Proposition 1 (and Proposition 3), because both the repayment and default payoffs vary with $s$, whereas in the case of $b$ the default payoff does not vary with $b$. The key to this Proposition is Conjecture 3, which states that, when the default set is non-empty for a given $(b, s)$, the available debt contracts and reserves choices associated with any $(y, p)$ in the default set imply trade surpluses at least as large as the excess of oil profits under repayment over those under default. Intuitively, the net resources that all available debt contracts and reserves choices can generate for consumption under repayment are at most the same as those obtained with the optimal reserves chosen under default.

**Proposition 5. If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.**

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $y_1 < y_2$, if $y_2 \in D(b, s)$ and $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ (where $b^1 \equiv b'(b, s, y_1, p)$, $s^1 \equiv s'(b, s, y_1, p)$ are the optimal choices of bonds and reserves under repayment with $y_1$ and $\tilde{s}^2 \equiv s^d(s, y_2, p)$ is the optimal reserves choice under default with $y_2$), then $y_1 \in D(b, s)$.*

This proposition shows conditions under which Proposition 3 in Arellano (2008) holds in this model. It shows that the sovereign has stronger default incentives at lower levels of non-oil GDP when the optimal trade balance under repayment with low $y$ is larger than the difference in optimal oil profits under repayment at that same low $y$ relative to those under default at a higher $y$. As noted earlier, this trade balance condition holds infrequently in the numerical solution but still the default incentives strengthen as $y$ falls in 94 percent of the state space. In the remainder 6 percent, defaults can occur even if $y$ does not fall.

**Proposition 6. If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall**.

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $p_1 < p_2$ and $p_2 \in D(b,s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where $b^1, s^1$ are the optimal bonds and reserves choices under repayment in state $(b, s, y, p_1)$ and $\tilde{s}^2$ is the optimal reserves choice under default in state $(s, y, p_2)$, then $p_1 \in D(b, s)$.*

This proposition shows sufficiency conditions under which the result in Proposition 5 with respect to non-oil GDP also applies to oil prices (namely that the sovereign has stronger default incentives when $p$ is lower). This Proposition assumes not only a sufficiently large trade balance but also that the oil reserves the sovereign chooses under default at a high $p$ are larger than those it chooses under repayment at a low $p$. This property holds in all of the state space of the calibrated model. The trade balance condition holds infrequently, but still in the numerical solution we found that default incentives strengthen as oil prices fall in all of the state space.

Summing up, the above theoretical findings indicate that the model preserves the standard properties of EG models with respect to debt and that these extend to oil reserves. In particular, repayment payoffs are nondecreasing in $b$ or $s$, the bond pricing function is increasing in $(b, s)$ and default incentives are generally stronger at lower $y$ or lower $p$. The theory also predicts that the default payoff is non-decreasing in $s$ and that optimal oil profits are increasing in existing reserves and decreasing in new reserves (i.e., increasing in extraction). Next, we use these results and the findings from the analysis in Appendix F for the model without default risk to provide an economic intuition of how oil extraction and debt compare in their effects on resources disposable for consumption under default and repayment, and to examine how the dependency of bond prices on debt, reserves and oil prices interact in the formulation of optimal extraction plans.

Consider first the effects of newly issued debt $b'$ and reserves choice $s'$ on resources available for consumption. Combining the constraints for the optimization problem under repayment yields:

$$c = y - A + p(s + \kappa) - ps' - e\left(s', s\right) + b - q\left(b', s', y, p\right) b', \tag{10}$$

where we replaced $x$ with $s'$ as an argument of $e(.)$. Note that, since $e(.)$ is increasing in $x$ and $x$ decreases with $s'$, $e(.)$ is decreasing in $s'$. The above expression shows key similarities and differences faced by the sovereign in the choice of $b'$ v. $s'$ for reallocating consumption intertemporally. By borrowing more (reducing $b'$), the government alters resources for current consumption according to the familiar debt Laffer curve of EG models.[16] Reducing $s'$ is akin to borrowing in that it increases resources for consumption by the amount by which $-(ps' + e(s', s))$ rises. In contrast with debt, however, there is no Laffer curve when "borrowing with reserves." Conditional on not hitting the feasibility boundaries of extraction, lower $s'$ always increases resources available for consumption.[17] Borrowing with $s'$ also differs from $b'$ in that it alters resources in the default state, by increasing them by the amount $-(h(p)s' + e(s', s))$ as $s'$ falls (i.e., at a lower rate than under repayment).[18]

Debt and reserves also have similarities and differences in how outstanding debt $b$ and existing reserves $s$ affect resources for current consumption. They are similar in that arriving at the repayment state with more debt (lower $b$) reduces resources by the amount $b$, while arriving with fewer reserves reduces resources by the amount $ps$. But they differ in that the debt repayment is non-state-contingent while the resources provided by $s$ vary with $p$. It is often noted in the sovereign debt literature that debt has poor hedging properties because it does not reduce the burden of repayment in "bad" states of nature (i.e., the repayment is uncorrelated with total income), but oil reserves are *worst* in this regard because the resources they provide correlate positively with oil prices (i.e., they provide fewer resources at lower $p$). Hence, viewing $b$ and $s$ as assets for hedging income fluctuations, reserves are inferior to debt. Moreover, the sovereign can default on $b$ to reduce the debt burden ex-post.

Qualitatively, debt and oil reserves have similar effects on conditional default probabilities and default risk. With regard to debt, Proposition 1 established that, as in EG models, default sets shrink with $b$ and as a result the conditional probability of default and default risk are non-decreasing in debt. Thus, $q(.)$ is non-decreasing in $b'$. On the side of oil reserves, Proposition 4 showed that default sets also shrink with $s$ and thus the conditional probability of default and default risk are non-decreasing in reserves. Thus, $q(.)$ is non-decreasing in

[16] When $b'$ is low so that default risk is low or zero, additional debt always gains resources for consumption, because bond prices fall little or stay at $q^*$, but as debt rises enough for default risk to reduce $q(\cdot)$ sufficiently, additional debt results in fewer resources for consumption.

[17] Note that $\partial c/\partial s' = -p - e_{s'}(s', s) = -(p - e_x(x, s)) < 0$ because $q^{Ond} > 0$ implies that $p - e_x(x, s) > 0$ for an interior solution of $x$ (see Appendices F and G). Hence, borrowing with reserves always increases resources for consumption because the asset price of oil is positive.

[18] In the default state, $\partial c/\partial s' = -h(p) - e_{s'}(s', s) = -(h(p)p - e_x(x, s)) < 0$ because $q^{Od} > 0$ implies that $h(p) - e_x(x, s) > 0$ for an interior solution of $x$ (see Appendices F and G).

$s'$. The rationale is that, although in the case of oil reserves the default payoff is increasing in $s$ instead of constant, the repayment payoff grows more than the default payoff as $s$ rises. Notice these are short-term or contemporaneous effects that refer to how country risk at date $t$ responds to the sovereign choosing to increase debt or reduce reserves at $t$.

Next we examine the interaction between sovereign risk and the sovereign's optimal oil extraction plans. For simplicity, so that we can conduct the analysis with familiar no-arbitrage conditions in sequential form, assume that we give to a sovereign who is committed to repay the model's equilibrium bond pricing function, $q\left(s_{t+1}, b_{t+1}, y_t, p_t\right)$, assuming that it is differentiable and satisfies other regularity properties.[19] The optimality conditions of the sovereign's problem yield the following no-arbitrage condition between the expected return on oil and the return on sovereign bonds (see Appendix F):

$$E_t\left[\tilde{R}^o_{t+1}\right] = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) - \frac{\operatorname{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{11}$$

In this expression, $R^b\left(s_{t+1}, b_{t+1}\right) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$ is the sovereign's gross return on bonds. Since we are assuming commitment, there is no default risk, but because $q(\cdot)$ is the equilibrium pricing function of the model with default, the planner internalizes that higher debt carries a higher interest rate than $R^*$ (since $q_b(\cdot) > 0$). Also, since debt is non-state-contingent, the Euler equation for bonds implies that at equilibrium $R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'(c_t)}{\beta E[u'(c_{t+1})]}$. The term $\tilde{R}^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{\left[q^O_t+q_s(t+1)b_{t+1}\right]}$ is the sovereign's gross return on oil inclusive of the financial benefit of higher reserves increasing resources available for consumption by rising the price of newly-issued debt. This rate of return can be rewritten as $\tilde{R}^o_{t+1} \equiv R^o_{t+1}\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$, where $R^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{q^O_t}$ is the "physical" return on oil and $\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$ is the financial return from higher reserves increasing $q(\cdot)$.

Condition (11) implies that the sovereign's optimal extraction and reserves plans are set so that the total marginal gross return on the oil it extracts exceeds the full marginal cost of its liabilities by a premium equal to $-\frac{\operatorname{cov}_t\left(u'(c_{t+1}),\tilde{R}^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$. This is akin to a standard equity

[19]We assume that $q\left(\cdot\right)$ is strictly concave and increasing in $b_{t+1}$ for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given $s_{t+1}$ (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 1), with $q\left(\cdot\right) = q^*$ for $b_{t+1} \geq 0$ and $q\left(\cdot\right) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q\left(\cdot\right)$ is also increasing and concave in $s_{t+1}$ for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$ and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given $b_{t+1}$ (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in $s_{t+1}$ and $\overline{s}(b_{t+1})$ is increasing in $b_{t+1}$.

premium, with the caveat that both the return on oil and the return on bonds include financial components. The former (latter) because of the effect of lower oil reserves (higher debt) reducing the price of sovereign debt (increasing the interest rate).

Appendix F examines the implications of condition (11) in two other scenarios: (i) permanent financial autarky (which is the same as the solution of the default payoff if $\lambda = 0$) and (ii) a constant bond price set at $q = q^*$ (which renders the model akin to a small-open-economy RBC model).

Under financial autarky, the model resembles a canonical closed-economy RBC model, in which condition (11) reduces to $E_t[u'(c_{t+1}) R_t^O] = u'(c_t)$. Hence, the planner uses oil reserves in a manner akin to capital accumulation in the closed-economy RBC model. Markets are incomplete because there are no assets to insure away the risk of the shocks to $p$ and $y$. Thus, the planner self-insures with reserves so as to facilitate consumption smoothing. There is also an implicit endogenous domestic real interest rate represented by the stochastic marginal rate of substitution in consumption. In the model with default, the planner has a similar incentive in the default state: being excluded from credit markets, it will use reserves to facilitate consumption smoothing, except that, because $\lambda > 0$ it assigns some probability to being able to re-enter the credit market.

In the case with $q = q^*$, condition (11) reduces to $E_t\left[R_{t+1}^o\right] = R^* - \frac{\mathrm{cov}_t\left(u'(c_{t+1}), R_{t+1}^o\right)}{E_t[u'(c_{t+1})]}$ which is analogous to the one obtained in small-open-economy RBC models for the excess return on physical capital. Markets are again incomplete, but here the sovereign has access to no-state-contingent bonds for self-insurance and consumption smoothing. Oil is a risky asset and carries a risk premium, but the returns on oil and bonds and the risk premium do not include the financial terms due to the effects of debt and reserves on the price of bonds. Moreover, since the risk premium is small (as is typical in RBC models), the model is close to yielding the Fisherian separation of the extraction and reserves plans from the savings and consumption plans that holds strictly without uncertainty. We show in Appendix F that the no-arbitrage condition without uncertainty becomes $R_{t+1}^o = R^*$ and yields a second-order difference equation in $s$ that determines the extraction and reserves decision rules independently of the bonds and consumption decision rules.

In the model with default, since default is infrequent quantitatively, when debt and/or reserves (and the history of oil-price and non-oil GDP shocks) are such that the probability of default becomes positive only in the distant future, the dynamics of oil extraction and reserves will display similar features. The model will behave in a manner similar to a canonical small-open-economy RBC model. One important prediction of this model is that, when oil prices are low, and therefore expected to rise due to mean-reversion, the planner has the incentive to cut extraction and increase reserves. To see this, use the definitions of the asset price of oil and oil dividends to rewrite the no-arbitrage condition $R^o_{t+1} = R^*$ as follows (assuming an internal solution for $x_t$ for simplicity):

$$\frac{p_{t+1} - e_x\left(x_{t+1}, s_{t+1}\right) - e_s\left(x_{t+1}, s_{t+1}\right)}{p_t - e_x\left(x_t, s_t\right)} = R^*. \tag{12}$$

Since $e(\cdot)$ is increasing in $x_t$ and decreasing in $s_t$, when $p_t$ falls relative to $p_{t+1}$, the planner reallocates extraction from $t$ to $t+1$ by increasing $s_{t+1}$. This is a key incentive that is also a work in the model with default, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As Propositions 4 and 6 show, the incentives to default at date $t$ are stronger when $p_t$ is low but, if the sovereign chooses not to default, the incentive to increase $s_{t+1}$ in response to lower $p_t$ *reduces* the default risk premium paid on bonds sold at $t$ (i.e., increases the price of newly issued bonds) because default sets shrink with $s$.

# 4 Quantitative analysis

## 4.1 Calibration

We calibrate the model to cross-country weighted averages of oil-exporting countries in the dataset described in Section 2. For $N$ countries indexed by $i$ and $T$ years indexed by $t$, a given variable $x_t$ is computed as: $x_t = \sum_{i=1}^{N} w_i x^i_t$, where the weight $w_i$ is time-invariant and equal to the average share of country $i$'s oil production in the total oil production of the $N$ countries over the $T$ years in the sample. We also compute the weighted averages for the subsample of countries that did not default (the nondefaulters set) and for the subsample of countries that did default (the defaulters set).[20]

[20] If for a given year and a given country there is no available data, we recompute the weights.

#### 4.1.1 Exogenous shocks

The model has two exogenous shocks, $y$ and $p$. To construct their stochastic processes we proceed as follows: First, we estimate a VAR with both variables for each of the 30 countries in our sample. The VAR model is standard and has this representation:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} + \begin{bmatrix} \rho_p & \rho_{yp} \\ \rho_{py} & \rho_y \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_p & \sigma_{yp} \\ \sigma_{py} & \sigma_y \end{bmatrix} \begin{bmatrix} \epsilon_{pt} \\ \epsilon_{yt} \end{bmatrix},$$

where $\epsilon_{pt}$ and $\epsilon_{yt}$ are mean-zero, i.i.d. random variables. The diagonal of the estimated covariance matrix of the innovations is a matrix with variances $\sigma_p^2$ and $\sigma_y^2$. The measure of $p$ is the real Brent price of crude oil, which is common to all countries, computed as the average of the daily nominal price (nominal spot BRENT) deflated by the US CPI. Our measure of $y$ is each country's GDP minus the country's oil rents in constant local-currency units. We take the logarithm of $p$ and demean it, so that $p$ represents percent mean deviations, and for $y$, we log the data and extract the cyclical component using the Hodrick-Prescott filter, so that $y$ represents percent deviations from trend.

Next, we compute the weighted average of the coefficients that are statistically significant. As noted above, the weights corresponds to shares in the average oil production between 1979 and 2014. We found that the coefficients $c_0$ and $c_1$ are statistically zero for all countries, with the exception of only one. $\rho_{py}$ is only significant for four countries and $\rho_{yp}$ for two. Their weighted average values across all countries are $0.054$ and $0.04$. The weighted average value for $\rho_{pp}$ is 0.901 and is significant for all 30 countries in the sample, while the weighted average value for $\rho_{yy}$ is 0.371 and is significant for 21 countries. Table 6 summarizes the results.

Table 6: VAR Process for Non-Oil Output and Oil Prices

| Parameter | Description | Value |
|---|---|---|
| $\rho_p$ | oil price auto-correlation | 0.90 |
| $\rho_y$ | non-oil output auto-correlation | 0.37 |
| $\rho_{py}$ | oil price non-oil output correlation | 0.05 |
| $\rho_{yp}$ | non-oil output oil price correlation | 0.04 |
| $\sigma_p^2$ | variance oil price innovations | 0.006 |
| $\sigma_y^2$ | variance non-oil output innovations | 0.007 |
| $\sigma_{py}, \sigma_{yp}$ | covariance non-oil output, oil price | $-0.002$ |

The model is solved using a standard value function iteration algorithm for sovereign default models over a discrete state space. For the exogenous processes $p$ and $y$, we construct a discrete approximation to a VAR with the coefficients shown in Table 6 using the approach proposed by Tauchen (1986) with a spanning factor of 2.15. This was chosen so as to match the standard deviations of $p$ and $y$. The realization vectors of $p$ and $y$ have seven and five values, respectively. For the endogenous state variables $b$ and $s$ we use a discrete grid with 61 and 54 nodes between values -0.6 and 0 for debt and 12 and 15.97 for oil reserves.

### 4.1.2 Structural parameters

The model has nine structural parameters. The preference parameters $\beta$ (discount factor) and $\mu$ (coefficient of relative risk aversion); the technology parameters $\kappa$ (oil discoveries), $\gamma$ and $\psi$ (elasticity and scale parameters of extraction costs); the financial parameters $r^*$ (risk-free rate), $\hat{p}$ (oil-price default penalty), and $\lambda$ (credit-market re-entry probability); and the autonomous spending coefficient $A$.

We set $\mu = 2$, a standard value in the literature. The risk-free rate is set to $r^* = 0.00775$, which corresponds to the average ex-post, US-CPI deflated yield on a 3-month U.S. Treasury bill for the 1955-2014 period (see Bianchi et al. (2016)). Total GDP is normalized so that $y + y^o = 1$, and hence the resource constraints can be interpreted as adding shares relative to GDP.

In order to separate the identification of the oil technology parameters from the analysis of sovereign default, we calibrate these parameters assuming that the technology is the same everywhere, particular across defaulters and nondefaulters. This allows us to calibrate the technology parameters using a variant of the model without default risk, akin to a small-open-economy RBC model in which oil reserves take the role of the capital stock. Moreover, since the equity premium is negligible in this class of models, and hence Fisherian separation of savings and investment nearly holds, we approximate the solution by solving the model under financial autarky with a discount factor set to represent the inverse of the relevant opportunity cost of capital $R^*$.[21]

We calibrate this autarky model using its deterministic steady state conditions as follows. First, from the law of motion of reserves, if follows that in steady state $x = \kappa$. Next, as we show in appendix G, the Euler equation for reserves (equation (G.19)) yields this steady-

[21]The autarky model can be solved with the same algorithm as the risk-free model by simply collapsing the grid of bonds to one element set to zero and redefining $A$ so that $A = 1 - c$, where by construction $A$ would include any steady-state debt service $-(\frac{r^*}{R^*})b$ present in the data. Using the data for nondefaulters, the weighted averages of the consumption- and debt-GDP ratios are $0.56$ and $-0.13$, respectively.

state condition:

$$\psi\left(\frac{\kappa}{s}\right)^{\gamma}\left[\gamma\left(\frac{\kappa}{s}\right)+r^*(1+\gamma)\right]=r^*, \tag{13}$$

using the assumption that $1/\beta = R^*$ and normalizing the steady-state oil price to $p^{ss} = 1$. Note that the ratio $\left(\frac{s}{\kappa}\right)$ defines the years of oil reserves remaining before they are exhausted and that in steady state the share of Gross Oil Output in GDP is $\frac{\kappa}{(y+\kappa-\psi\left(\frac{\kappa}{s}\right)^{\gamma}\kappa)}$.

The calibration strategy is to: (a) set $\gamma$ to match the observed standard deviation of oil extraction (13.1%) in the stochastic solution of the autarky model; (b) given $\gamma$, impose on the above Euler equation the expected years of reserves estimated from the data (70.06) to solve for $\psi$, and (c) impose $\gamma$, $\psi$ and the data estimate of the share of Gross Oil Output in GDP (33.5%) on the definition of this share to solve for $\kappa$. These three data targets correspond to weighted averages of the nondefaulters in our sample.

We then follow an iterative procedure that starts with a guess for $\gamma$ and solves for the associated values of $\psi$ and $\kappa$ as indicated in (b) and (c). Then, we solve the stochastic autarky model to compute the standard deviation of oil-extraction, the mean of $\left(\frac{s}{\kappa}\right)$, and the mean of the ratio of oil rents to GDP, and iterate until these three model moments get as close as possible (up to a convergence criterion) to their data counterparts. With $\kappa = 0.3325$, $\gamma = 1.56$ and $\psi = 124.6544$, the oil- extraction standard deviation is 13.1%, the years of reserves $\frac{s}{\kappa} = 70.17$, and the Gross Oil Output in GDP share is 33.7%, all three very close to the data moments.

Given the values of the technology parameters, we switch to calibrate the baseline model with sovereign default. The annual probability of reentry is $\lambda = 0.332$, based on findings by Richmond & Dias (2007) who found a median period of financial exclusion of three years after default in a sample of 128 sovereign defaults during the 1980-2005 period. The mean interest rate is set to $r = r^* + spread$, where the $spread$ is the weighted average of the country spreads for the period 1979-2016.[22] The value of the $spread$ for the full sample is 707 bp.

The values of $b$ and $c$ are set to $-22.45\%$ and $58.88\%$, respectively, which correspond to the 1989-2016 weighted means of the GDP ratios of external debt and private plus public consumption, respectively, including all countries in our sample. To set the value of $A$, we

[22]For the period 1998-2016 we use JP Morgan's EMBI+GSS spreads data. Since these data start in 1998, for the 1979-1997 period we extrapolate the spreads measure by first regressing the EMBI data on the Institutional Investor Index in the common sample for the 1998-2016 period, and then use this regression and observed pre-1998 III values to estimate EMBI spreads for 1979-1997.

impose the assumption that total GDP is normalized to 1 in the resource constraint evaluated at the deterministic steady-state, so that $A = 1 + (r/R)b - c$, which yields $A = 0.3936$. Similarly, we set the mean of non-oil GDP to $E[y] = 1 - y^o$ where $y^o = 0.2069$ is the weighted average of the ratio of oil rents to GDP in the data including all countries (so that $E[y] = 0.793$).

Finally, the values of $\beta$ and $\hat{p}$ are jointly determined so that the stochastic baseline model solution matches the debt-to-GDP ratio ($22.45\%$) and the default rate ($1.14\%$) from the full dataset, including defaulters and nondefaulters.[23] These are weighted averages of the individual debt ratio and default frequency of each country, respectively. The two parameter values are set following an iterative procedure similar to the one used for the technology parameters: Start with a guess for $(\beta, \hat{p})$, then solve the model and simulate it to compute the model's mean debt-to-GDP ratio and default rate, and iterate until the model and data moments differ by a convergence criterion. This procedure yields $\beta = 0.82$ and $\hat{p} = 0.64$, and with the complete parameterization the mean debt ratio is $22\%$ (very close to the data) and the default rate is $1.19\%$ (just slightly above the $1.14\%$ in the data). Note, however, that since oil GDP is endogenous and $\kappa$ was calibrated separately, the model yields $E[y^o] = 0.221$, slightly above the $0.206$ in the data, and $E[GDP] = 1.0145$, just a little above its normalized deterministic steady state.

Table 7 compares the moments used as targets for the calibration with their counterparts produced by the model and Table 8 lists all the parameter values. There are two targeted data moments (average external debt to GDP and the default rate) which the model should match closely. The rest of the moments shown in the table are not targeted, and as such, can be contrasted with those of the data to gauge the model's ability to replicate them. To compute the business cycle moments, the actual data were logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$, while for model-generated data we do not detrend (since the model is stationary by construction) and report coefficients of variation instead of standard deviations so that the measures of dispersion in model and actual data are both in percent. The variability of oil extraction in the model is $12\%$, close to its data counterpart ($12.2\%$). The model falls short of replicating oil reserves ($43$ years in the

[23] All nondefaulters enter in the weighted sum that yields this estimate of the aggregate default rate with a zero default rate and their corresponding weight in total oil production. Including only defaulters, the default rate would be $2.2\%$.

model v. 62 years in the data). The low discount factor (which is not uncommon in sovereign default models) incentivizes the planner to reallocate resources towards the present, but the lack of commitment limits its capacity to to do so via borrowing in bonds and thus reduces reserves in the long run.

Table 7: Data vs Model Moments

| Description | Data | Model | | |
|---|---|---|---|---|
| | | Benchmark | Constant Extraction | Risk Free |
| Average External Debt to GDP | 0.225 | 0.229 | 0.276 | 0.517 |
| Default Rate | 1.14% | 1.19% | 1.08% | 0% |
| Standard Deviation of Oil Extraction | 0.122 | 0.120 | 0.000 | 0.123 |
| Average Reserves (in years) | 62 | 43 | 42 | 42 |

Estimates of the proven reserves for the average oil exporting country correspond to those of the US Energy Information Administration.

Table 8: Parameter values

| Parameter | Description | Value |
|---|---|---|
| $\beta$ | discount factor | 0.82 |
| $\mu$ | risk aversion coefficient | 2.00 |
| $q^*$ | risk-free debt price | 0.99 |
| $\hat{p}$ | oil-price default cost threshold | 0.64 |
| $k$ | discovery rate | 0.33 |
| $\lambda$ | re-entry probability | 0.33 |
| $\gamma$ | extraction costs curvature | 1.56 |
| $\psi$ | extraction costs scale | 124.6 |
| $A$ | autonomous spending | 0.40 |

## 4.2 Model versus Data

We start the analysis of the quantitative results by comparing the model-generated data with the actual data in three key dimensions: long-run moments, default-event dynamics, and the oil reserves-sovereign risk co-movements.

**Data Moments** Columns (1)-(2), (5)-(6), and (9)-(10) of Table 9 compare the long-run moments from the data with those produced by the baseline model. The Table includes additional columns listing the moments produced by two variants of the model, one with constant oil extraction (to examine the relevance of endogenizing oil production) and a risk-free model where the sovereign is committed to repay (to assess the role of default risk). The results for these two model variants are discussed later in this section.

We report the long run moments of both the data and the model relative to disposable income, where disposable income is defined as total GDP minus $A$ (autonomous spending), and its variability in the data corresponds to the standard deviation of the Hodrick and Prescott cyclical component. In the model the variability of disposable income corresponds to the coefficient of variation. As can be seen from Table 9 the model overestimates the variability of disposable income by a bit but does very well at generating the relative variability of total GDP, even though it underestimates the variability of gross oil output by some. The relative volatility of extraction is lower in the model, but the relative volatility of consumption and the spread are higher.[24] The model underestimates the variability of the trade balance, but does relatively well in terms of the variability of debt (note that the variability of deb to GDP for each column is the number that corresponds to Debt/GDP times the one for Disposable Income, so the variability of debt to GDP is $0.14$ in the data and $0.10$ in the benchmark).

In terms of the model's correlations with disposable income, we see that the model does particularly well at matching that of extraction, while it overestimates that of gross oil output, total GDP, and consumption. This suggests that there is more consumption smoothing going on in the data than in the model. The trade balance has a much lower correlation with disposable income in the model than in the data and debt to GDP and the spread are more counter-cyclical in the model.

For gross oil output, total GDP, disposable income, extraction, and consumption, the model overestimates the correlation with the price of oil. The spread is less negatively correlated with the price of oil in the model, and even though debt to GDP has a more negative correlation in the model than in the data, the model does pretty well in this dimension. Finally, in the data the trade balance is positively correlated with the oil price, while in the model it has a negative correlation. This is because in the model default episodes (sovereign risk) have a negative relationship with oil prices. In other words, at higher oil prices the probability of default goes down, and the sovereign can sustain more debt, generating a negative trade balance, while at low prices their ability to acquire debt goes down, generating positive changes in the trade balance.

The high volatility of consumption is an unappealing feature of these results, and to understand the intuition behind it, it is important to compare the results of our benchmark

[24]For the spread we use the EMBI spread calculated as described in the previous subsection

model with three model variants: the model where oil extraction is exogenous and constant (constant extraction), the model where there is no default (risk free), and the model where the country is in financial autarky. The constant extraction model is like the one described in Section 3, but the sovereign cannot endogenously decide how much oil to extract, oil extraction is equal to an exogenously given amount that is constant. The risk free model, is like the one described in Section 3, but the sovereign has to commit to repaying its debt, there is no default in equilibrium. And the financial autarky model is such that the sovereign can not acquire any debt with the rest of the world, and can only use its output (both oil and non-oil) to finance its spending.

We can see from Table 9 that the relative coefficient of variation of consumption in the risk free model is lower than that in the benchmark model (0.98 versus 1.07). This tells us that oil reserves are playing their expected role of facilitating consumption smoothing because in the risk free model reserves and bonds are savings vehicles and the lack of repayment commitment does not hamper credit access, while in the variants with either lack of commitment (baseline model) or both lack of commitment and inability to use reserves to smooth (constant extraction model), consumption smoothing is hampered and its relative variability is higher. In addition, in the baseline model, the planner hits (albeit infrequently) the upper bound of the bonds grid, preventing it from taking a positive bonds position and thus working as an implicit savings constraint that contributes to hamper consumption smoothing.

We can also determine that the planner is using oil reserves optimally to smooth consumption in the baseline model because the boundaries of the grid for reserves never bind. The planner adjusts reserves taking into account the interaction between debt, debt prices, and reserves when aiming for the consumption path that maximizes private utility. The lack of commitment limits borrowing capacity but this does not lead the planner to reduce reserves to its lowest feasible level. Similarly, even when the sovereign hits the debt limit in the risk-free model, it still does not choose to hit the boundaries of the grid of reserves.

In the case of the constant extraction model, in which reserves cannot be used to smooth consumption but default risk remains, the reduced ability to smooth consumption relates to a feature of EG models identified by Chatterjee & Eyigungor (2012): When debt is large relative to output, a change in the bond price implies large changes in consumption given that the sovereign must refinance all of the debt at the new price in one period. Because of the ladder-like shape of the equilibrium price of bonds, changes in bond prices can be large.

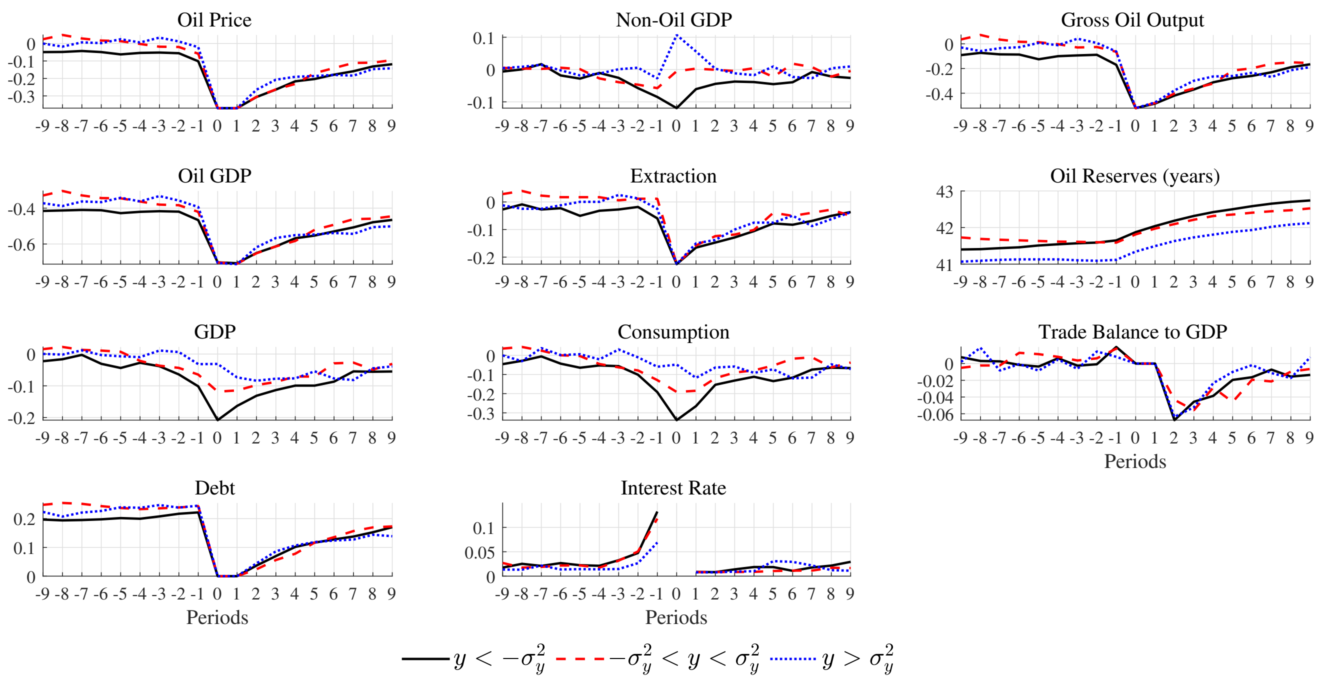
Table 9: Long-run moments - Data v. Model

| | Variability relative to DI | | | | Correlation with DI | | | | Correlation with Oil Price | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| | Data | Benchmark | Constant Extraction | Risk Free | Data | Benchmark | Constant Extraction | Risk Free | Data | Benchmark | Constant Extraction | Risk Free |
| Gross Oil Output | 2.4 | 1.74 | 1.16 | 1.73 | 0.51 | 0.69 | 0.72 | 0.66 | 0.34 | 0.96 | 1.00 | 0.96 |
| Total GDP | 0.67 | 0.61 | 0.61 | 0.61 | 0.62 | 1.00 | 1.00 | 1.00 | 0.12 | 0.72 | 0.72 | 0.69 |
| Disposable Income (DI) | 0.10 | 0.17 | 0.17 | 0.16 | 1.00 | 1.00 | 1.00 | 1.00 | 0.12 | 0.72 | 0.72 | 0.69 |
| Extraction | 1.20 | 0.70 | 0.00 | 0.75 | 0.52 | 0.53 | 0.00 | 0.50 | 0.04 | 0.75 | 0.00 | 0.77 |
| Consumption | 0.48 | 1.07 | 1.08 | 0.98 | 0.34 | 0.97 | 0.91 | 0.99 | 0.12 | 0.73 | 0.74 | 0.71 |
| Trade Balance/GDP | 0.88 | 0.26 | 0.28 | 0.05 | 0.39 | 0.03 | 0.04 | 0.20 | 0.19 | −0.15 | −0.16 | −0.03 |
| Debt/GDP | 1.41 | 0.62 | 0.72 | 0.33 | −0.07 | −0.42 | −0.44 | 0.98 | −0.61 | −0.79 | −0.80 | 0.66 |
| Spread | 6.29 | 15.74 | 16.62 | 0 | −0.09 | −0.28 | −0.25 | 0.00 | −0.46 | −0.14 | −0.10 | 0.00 |

* Actual data are for the 1979-2016 period, logged and HP-detrended, except for the TB/GDP, Debt/GDP ratios and the EMBI, which is in levels (basis points). Model data are not detrended because the model is stationary by construction.

Variability in the data of the Disposable Income (DI) corresponds to the standard deviation of the HP cyclical component, and in the model corresponds to the coefficient of variation. The rest of the variables report the relative variability to DI.

Figure 8: Different types of defaults in the model




**Default Event Dynamics** We evaluate the model's performance at explaining the observed default event dynamics by constructing default event windows using data from a simulation of the model with 10,000 periods. All the default events are identified and sorted into the same groups as in the default event analysis from the data shown in Figure 7. Figure 8 shows the results. Unlike in the data, in the model defaults only occur when oil prices drop. But interestingly the model is consistent with the data in predicting that defaults can occur with high or low non-oil GDP. The black line shows the event analysis for the case when $y$ is below trend (by one standard deviation or more) which constitutes $46\%$ of all default episodes, the dotted blue line shows the case when $y$ is above trend (by one standard deviation or more) and it represents $17\%$ of all default episodes, and the dashed red line are default episodes that occur when non-oil GDP is within one standard deviation of its trend and accounts for $37\%$ of all default episodes.

When we look at the behavior of total GDP at the time of default (between t=-1 and t=0), we can see that in two cases it is decreasing (the black and dashed red lines) and in the case of the dotted blue line it is increasing. This means that in the model, $17\%$ of defaults occur in good times and $83\%$ of defaults occur in bad times. These results show that in terms of non-oil GDP our model is as versatile as the data, and that it can generate defaults in good and bad times.

We believe that an important result to highlight from Figure 8 is that in all three cases oil reserves increase at the time of default. Remember that one of our main results from the empirical part is that oil reserves are positively correlated with sovereign risk. However, our empirical results do not speak about causality. What this result about default episodes is saying is that a default episode generates an increase in reserves because it is optimal to decrease extraction as present oil prices are lower than expected, saving oil for future sales. This mechanism can explain the positive relationship between oil reserves and sovereign risk that we observe in the data.

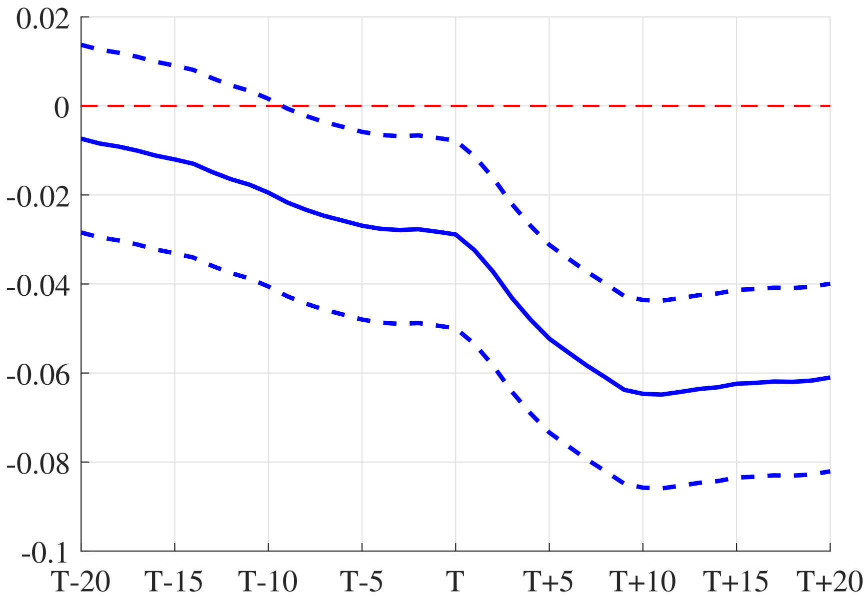
At the same time, these results reinforce the observation that a larger stock of oil reserves is positively associated with sovereign risk in the long run, while oil production is negatively associated with sovereign risk as shown in our empirical results of Section 2.2 (see Table 2). In the average episode before the default, GDP is trending down and the interest rate is going up. After the default occurs, oil reserves go up, as does the interest rate. This we believe alludes to the fact that having a large stock of the real asset increases the value of autarky making default more appealing, while oil production is negatively associated with sovereign risk because it increases the ability of the sovereign to repay its debt.

In the next subsection we explore these mechanisms further, and test more directly the ability of our benchmark model generated data to replicate the empirical facts of Section 2.2.

**Oil Reserves and Sovereign Risk** The dynamic panel analysis of Section 2 produced the interesting finding showing that higher oil reserves *increase* sovereign risk in the long run. We explore next whether the model we proposed can rationalize this result. We do this first by studying the cross-correlation function of country risk and oil reserves (i.e., the correlation between $q$ in period $t$ with $s$ in periods $t - 20$ to $t + 20$) and second by estimating the same dynamic panel regression as in Section 2 but with the model-generated data.

The cross-correlation function of bond prices with oil reserves plotted in Figure 9 shows that current sovereign bond prices have a more negative correlation with future oil reserves than with current or lagged oil reserves. Prices are nearly uncorrelated with reserves at the twentieth-year lag and converge to a statistically significant (albeit small) correlation of about -0.06 five years into the future and beyond. This indicates that date-t default spreads are positively correlated with *future* oil reserves, in line with our empirical finding showing that higher reserves in the future increase sovereign risk in the present.

Figure 9: Cross-Correlation Function of Bond Prices and Oil Reserves in the Model




To estimate the dynamic panel regression on model-generated data, we simulate the model for 10,000 periods and bootstrap thirty-year windows to run the same regression shown in Table 2. Three caveats of this exercise are worth noting. First, as explained earlier, data limitations led us to use as a measure of country risk for the actual-data regression the Institutional Investor Index, for which higher III values denote lower country risk. For the model, however, we use the model-generated data on sovereign bond prices (which also rise as country risk falls). Second, in the model the market of sovereign bonds shuts down when a default occurs and effectively bond prices go to zero but in the data the III (and also bond prices) has values even in default events. Thus, for the model regression we exclude default events and we cannot include the default dummy as a regressor. Third, in the model regression we cannot control for discoveries because $\kappa$ is constant.

The main goal of this exercise is to determine whether the qualitative properties of the co-movements identified in the data (as represented by the signs of the regression coefficients) match those predicted by the model. As highlighted in Section 2, the main results from the dynamic panel estimation are that, in the long-run, sovereign risk falls as oil GDP rises but increases as oil reserves rise. The results from estimating the model's dynamic panel, reported in Table 10, shows that the model yields the same qualitative co-movements. Oil production and non-oil GDP have positive and significant long-run coefficients, while oil reserves have a negative and significant long-run coefficient (at the $5\%$ level).

The results that higher oil production and non-oil GDP reduce country risk are easy to rationalize using the analytic findings of Section 2: Higher $y^O$ or higher $y$ increase a country's ability to repay their sovereign debt and also weaken default incentives (if the trade balance under repayment is sufficiently high, see Proposition 5). On the other hand, Proposition 4 shows that default sets shrink in $s$, which would suggest that higher reserves should reduce country risk. Indeed, the short-run effect of reserves on bond prices is positive in the model's dynamic panel (see Table 10). The analytic result assumes, however, permanent exclusion of credit markets after a default, iid shocks and no oil-price default penalty, and it also keeps $(y, p, b)$ constant. In the full model solution, which relaxes all these assumptions, higher oil reserves increase sovereign risk in the long-run because the planner expects that they will increase the value of autarky relatively more than that of repayment. In the event of default, the planner substitutes away from using debt to using oil reserves (a financial asset for a real asset) in order to smooth consumption, and higher reserves strengthen the sovereign's capacity to "borrow from reserves."

### 4.3 The role of the oil sector

Next we use the quantitative results to assess the role of the oil sector in the model. We compare the baseline model results with those produced by the constant-extraction model (i.e., with oil output as an endowment) and the risk-free model (i.e., with endogenous oil extraction but with a government committed to repay its debt). The first comparison shows the importance of having the ability to choose oil extraction optimally for the sovereign's choices of debt issuance and default. In the benchmark case the sovereign has the ability to smooth consumption by either increasing debt or by selling more oil abroad. If the extraction decision is not endogenous and furthermore, its exogenously set to a certain value, the sovereign looses the ability to optimally evaluate the trade off between using debt and oil to finance spending and smooth consumption. The second comparison shows the extent to which de-

Table 10: Dynamic Fixed Effects Regression Results for Bond Price Using Model-Generated Data Index

| | $\Delta$ Bond Price |
|---|---|
| **Convergence Coefficients** | |
| Bond Price (-1) | -0.684*** |
| | (0.011) |
| **Short-Run Coefficients** | |
| $\Delta$ Oil Production | -0.016*** |
| | (0.006) |
| $\Delta$ Non-Oil GDP (LCU) | 0.002 |
| | (0.004) |
| $\Delta$ Oil Reserves | 1.717*** |
| | (0.191) |
| $\Delta$ Ext. pub. debt to GDP | -0.012* |
| | (0.007) |
| **Long-Run Coefficients** | |
| Oil Production | 0.050*** |
| | (0.006) |
| Non-Oil GDP (LCU) | 0.087*** |
| | (0.004) |
| Oil Reserves | -0.015** |
| | (0.007) |
| Ext. pub. debt to GDP | -0.008* |
| | (0.004) |
| Constant | 0.104*** |
| | (0.022) |
| Observations | 8691 |

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

fault risk matters for oil extraction and reserves decisions by showing how the results would vary if the sovereign makes oil extraction plans without default risk. This matters because if the sovereign is committed to repaying its debt, it can sustain more debt in equilibrium, and this affects the optimal oil-debt trade off that we observe in the benchmark model.

Table 7 shows how the moments used as targets for the benchmark calibration differ across models. The mean debt ratio rises to 51.7% in the risk-free model but it fluctuates very little (see Table 9 for variability), because with a discount factor of $\beta = 0.82$ the planner has a very strong incentive to borrow and thus hits this economy's ad-hoc debt limit 88% of the time. This should incentivize the planner to substitute debt for oil reserves as a vehicle for consumption smoothing and thus build-up precautionary reserves for self insurance. The variability of oil GDP and consumption does fall relative to the baseline model (for the latter, see Table 9) but mean reserves hardly change. This occurs because the rate of return on oil reserves is not constant, like the risk-free rate, but changes as extraction costs respond to changes in reserves. Although in principle the risk-free model is akin to a small-open-economy RBC model, the low discount factor pushing the debt to its lower bound makes it more similar to a closed-economy RBC model with oil reserves taking the place of the capital stock. The ability to smooth with reserves is hampered by their endogenous return (effectively, the real interest rate of this economy is endogenous). In the constant extraction model (with default risk), the mean debt ratio increases about 5 percentage points relative to the benchmark model, which indicates that lacking the ability to use reserves to support consumption during periods of exclusion enhances the sovereign's borrowing capacity overall and it also reduces slightly the frequency of defaults (about 10 basis points)–we elaborate more on these two results later on. Mean oil reserves and oil-GDP variability remain about the same as in the benchmark.

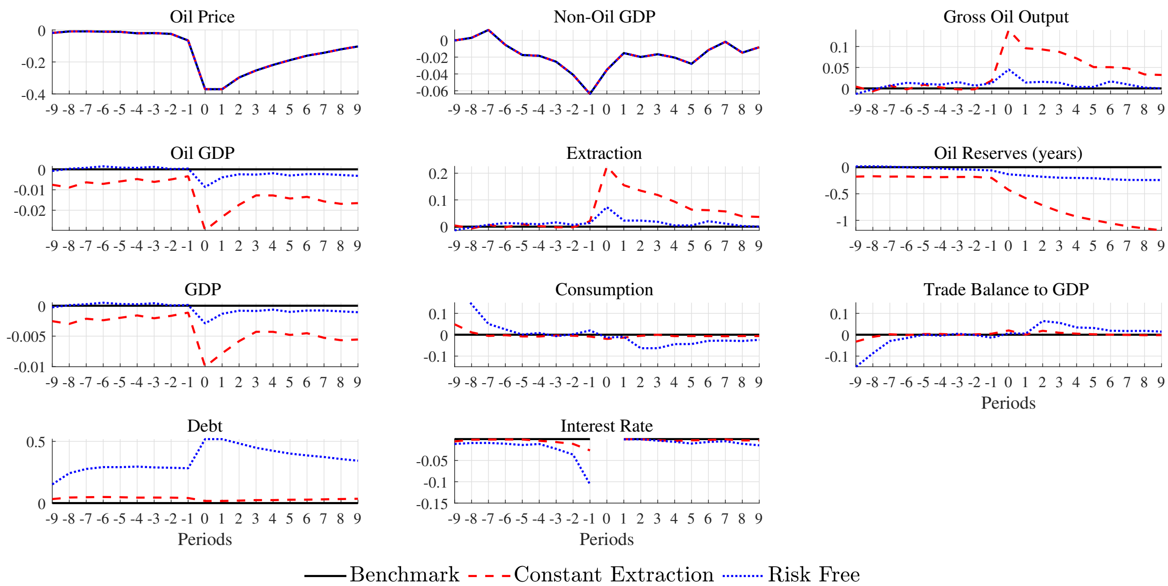
Table 9 compares long-run cyclical moments across the three models. As noted above, consumption is less volatile in the risk-free model, although not by much because the low $\beta$ makes the planner hit the debt limit very often and the extraction costs hamper the ability to adjust reserves to smooth consumption. For the same reason, debt and the trade balance fluctuate much less in the risk free model than in the other two, consumption is more correlated with disposable income, and the trade balance is more procyclical and less negatively correlated with oil prices. Several of the other moments are similar between the benchmark and risk-free model, particularly those for gross oil output and total GDP, suggesting that, even tough Fisherian separation of the consumption/borrowing decisions from the choices of reserves and extraction does not hold in both the baseline and risk-free models (in the former because of default risk and in the latter because of the binding debt limit), the resulting distortions on the Euler equation for oil reserves do not result in significant differences in disposable income and oil-price correlations of gross oil output and total GDP. The same is true for the constant extraction model, which suggests that Fisherian separation also holds approximately in terms of the moments that characterize business cycles in total GDP and disposable income when default risk is introduced vis-a-vis once is removed. Keep in mind that these results pertain to business cycle co-movements over the long-run. The similarity of some cyclical moments does not imply that default risk and endogenizing oil extraction do not have significant interactions. Even in the long run, we have found that endogenizing extraction decreases the mean debt ratio and reduces the average spread, and that higher oil reserves increase default risk. Moreover, default risk and endogenous oil extraction also affect the dynamics of default events, as we show next.

We now compare the dynamics of default events across the three models. To do so, we use again the ten-thousand period simulation of the baseline model, identify the default episodes that it produces, and take 19-period event windows centered on the period in which the default occurs. In order to make the paths of the constant extraction and risk free models comparable to the benchmark, we take the realizations of $p$ and $y$ in each of the 19-period windows of each default event episode of the benchmark model, as well as the initial debt and reserves of each default episode and feed them into the policy functions of the model with constant extraction and the risk free model to recover the paths followed by the different variables in those two specifications. The mean of all the default episodes in each model relative to the benchmark model path is represented in Figure 10. As such the solid black line is always constant, the red-dashed line represents the behavior of the model with constant extraction relative to the benchmark model, and the dotted-blue line represents the behavior of the risk-free model relative to the benchmark model.

Figure 10 shows that even if the path for oil prices and non-oil GDP is the same in the three models, the behavior of extraction, reserves, oil GDP, total GDP, debt, interest rates, consumption, and the trade balance is not. At the moment of default oil extraction in the constant extraction model is higher than in the benchmark because it cannot be adjusted (in the benchmark because defaults happen at low oil prices, the sovereign prefers to reduce extraction today in the hopes that prices go up in the future). For this same reason reserves are lower in the constant extraction case. However, the higher relative extraction is not high enough and oil GDP and total GDP are lower in the constant extraction case. Note that this is because oil extraction costs are higher (gross oil output is higher), under the constant extraction model because of the higher extraction, and the increase in revenues is not enough to cover the increase in the extraction cost.

Furthermore, something that is not obvious from Figure 10, but is an important difference between the benchmark model and the constant extraction model, is that conditional on the same path for oil prices and non-oil GDP, the constant extraction model defaults only $85\%$ of the times compared to the benchmark. This is because under constant extraction the oil price has to drop $40\%$ from trend to generate a default, if prices are below trend and drop another $20\%$ the constant extraction model doesn't generate a default while the benchmark

Figure 10: Before and after default episodes: benchmark, constant extraction and risk free model




Note: All variables are reported relative to the Benchmark model, except for the Oil Price and Non-Oil GDP which are plotted relative to their long-run average

does. The default in the benchmark is accompanied by a sharp drop in extraction (due to the lower prices) and an increase in oil reserves. This shows once more that having endogenous extraction in the model is important to generate the positive relationship between oil reserves and sovereign risk that we observe in the data.

The behavior of the risk free model is also different. In that setup there is no default in equilibrium, so debt is between 25% and 50% higher than in the benchmark model while interest rates are lower. Also, even though extraction drops at the time of default it doesn't drop as much as in the benchmark model, so in relative terms it is higher (as shown in Figure 10), but again, as in the model with constant extraction, even though gross oil output is higher, so are extraction costs, such that oil GDP and total GDP are lower.

To summarize, in this subsection we have shown that although having exogenous extraction might not change the model generated data moments drastically, it does matter for the dynamic of macroeconomic variables, for default rates, and how default episodes are triggered.

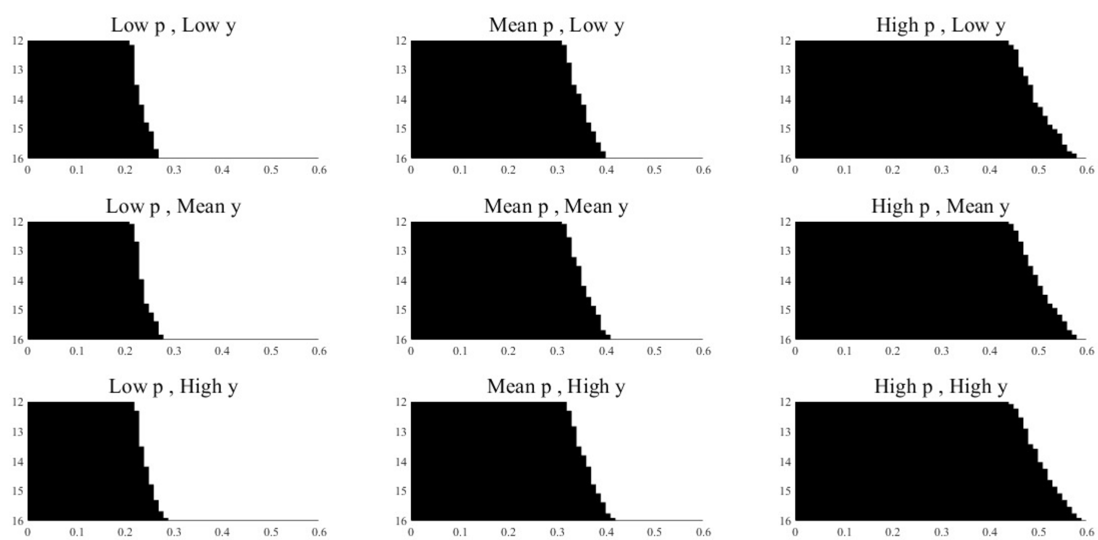
### 4.4 Default sets and default costs

Given that this model is different from the usual sovereign default model in the literature because default sets are endogenous, in this subsection we look at default sets and default costs.

**Default sets** Figure 11 shows the default sets. In the subplots prices are increasing to the right and non-oil GDP is increasing down. In the $x$ axis we have debt increasing to the right, and in the $y$ axis we have reserves increasing downward. The light area is the region of default and the dark area is the repayment region. As can be seen from the figure, the default region is decreasing in prices and increasing in non-oil GDP. Higher oil prices represent a higher ability to repay, and higher non-oil output increases the value of financial autarky.

These results are in line with those in the previous subsection, where we illustrate the default episodes, and showed that they occur when oil prices fall, and are more common when non-oil output increases.

Figure 11: Default Sets

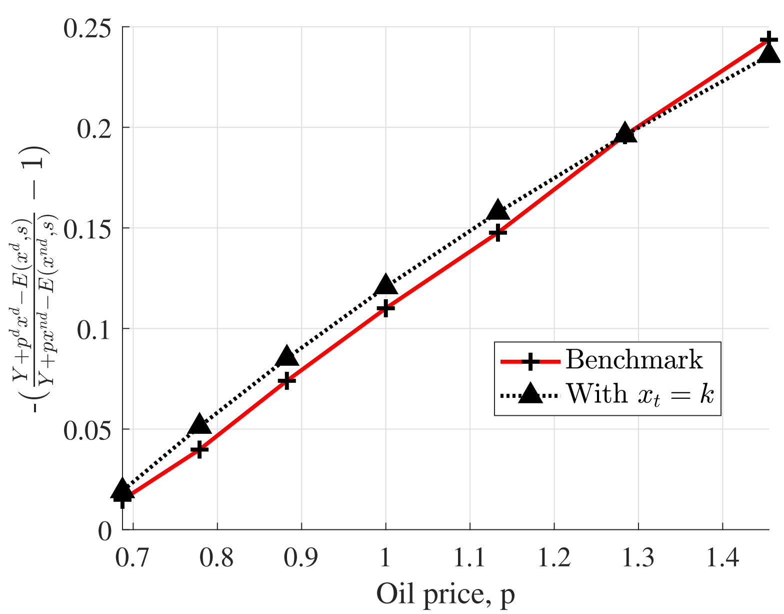



**Default costs** In order to prevent default from happening too often and allowing models to sustain debt, the literature introduces an adhoc default cost on output. This cost ranges from just loosing a percentage of output, to more complicated linear or non-linear loss functions. In the more involved cases, this cost is zero for output values below a certain threshold and is increasing with output above that threshold as to induce default in bad times.

Recall that total GDP in the model is $y^T = y + px - e(x, s)$, and in a state of default the oil revenue becomes $h(p)x$. As mentioned in Section 3 this default cost aches to a trade penalty that is imposed when default takes place, such that oil revenues are reduced. In order to compute the total cost of default on GDP, we compute total GDP under default ($y^{Td} = y + p^d x^d - e(x^d, s)$, where $p^d = h(p)$) as a fraction of what the GDP would have been with the same realizations of the exogenous variables under repayment ($y^{Tnd} = y + px^{nd} - e(x^{nd}, s)$).

Figure 12 shows the results for the benchmark model (red solid line), and for the model with constant extraction (dotted black line), for the case when oil prices vary and we set debt, reserves, and non-oil aoutput to their mean values. As we can see default costs are increasing in oil prices, and they range between $1.6\%$ and $22\%$ in terms of the GDP that could be attained under repayment. Even though these default costs might seem large, they are not according to the sovereign default literature. For example, the default cost in Arellano (2008) ranges between $0$ and $30\%$ of GDP.

Figure 12: Default Costs in terms of GDP




Note that our default cost is not only comprised of an exogenous adhoc component, but that it also has an endogenous component because effective oil prices affect extraction decisions, and hence, the extraction cost, and oil GDP. The adhoc component of our default cost, amounts to a progressive tariff on oil prices that ranges between $8\%$ and $56\%$. In other words if a default were to be triggered by the lowest realization of the oil price, then the effective oil price faced by the sovereign would be $8\%$ lower, and if a default is triggered by the highest realization on the oil price, then the effective oil price faced by the sovereign would be $56\%$ lower. However, the default cost in terms of oil GDP (not just the oil price) ranges between $15\%$ and $74\%$ because of the effect that lower prices have on extraction and extraction costs. These costs in terms of oil GDP translate to the costs in terms of total GDP depicted in Figure 12.

Hence, our model has the advantage that default costs are not fully exogenous but they have an endogenous component, unlike the rest of the sovereign default literature.

## 5 Conclusion

We have shown, that being a resource rich country—and more specifically—having oil, has two different effects on country risk. First, in the short-run it decreases country-risk because it increases a countries ability to repay and second, in the long-run, having a large stock of oil reserves increases country risk as it increases the value of autarky for the country as it helps them withstand exclusion from international financial markets.

We develop an off-the-shelf sovereign default model with oil extraction and show that the model is capable of generating the same relationships that are present in the data.

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# Appendix for
# Resource Curse or Blessing? Sovereign Risk in Resource-Rich Emerging Economies

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# Contents

# A Data

We have collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, total public debt, total external public debt, net foreign assets, default episodes and country risk, for the thirty largest oil producing emerging economies as of 2010. Those thirty countries are Saudi Arabia, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam.

As an indicator of country risk we use the Institutional Investor Index (III from now on). The III country credit rating, is a measure of sovereign debt risk that is published biannually in the March and September issues of the Institutional Investor magazine. It is also commonly known as the Country Credit Survey. More specifically, the III is an indicator used to identify and measure country risk, where country risk refers to a collection of risks related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. We have biannual data for the 1979-2014 period. The index is based on information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents have graded each country on a scale of zero to 100, with 100 representing the least likelihood of default. Respondents responses are weighted according to their institutions' global exposure.

The data on oil reserves, oil production, oil net exports (thousands of barrels per day), and oil prices (Brent crude oil, USD per barrel) is from the US Energy Information Administration (EIA) from 1980 to 2014. For reserves, we used proved reserves. For oil prices we use the real price by deflating the Brent spot price FOB with the US CPI index for all urban consumers all items in US City average, seasonally adjusted (1982-1984=100).

Total public debt data comes from the International Monetary Fund's Historical Public Debt Database (HPDD). We have information, covering 1971-2015 period, for Gross Government Debt. Total public external debt data is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, Oman, Malaysia and Syria.

We use the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane & Milesi-Ferretti (2007) to obtain information on net foreign asset positions. It contains data for the 1970-2015 period and for 188 countries (including those in our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i) the accumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between accumulated flows of "debt assets", and the stock of debt measured by the World Bank (or the OECD).

Default data is from Borensztein & Panizza (2009) for the 1979-2004 period. We include sovereign defaults on foreign currency bond debt and foreign currency bank debt. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary. We use the updated and extended version default data from Reinhart & Rogoff (2010) dataset for the 2005-2014 period. A default is defined as an external sovereign default crisis or a restructuring of external debt.

# B Institutional Investor Index & Sovereign Risk Measures

In this section, we show that the Institutional Investor Index (III) is a robust measure of sovereign risk by showing that it is highly correlated with other measures of sovereign risk. We also explain how we use the III to chain the Emerging Markets Bond Index (EMBI) backwards to be able to use it to calculate the average and standard deviation of the spread used in Section 4.

## B.1 Moody's and Fitch Credit Ratings

Credit ratings by agencies such as Moody's and Fitch are commonly used measures of sovereign risk. These agencies assign risk based on rating symbols. Tables B1 and B2 provide brief descriptions of what each symbol signifies about credit risk. Table B3 provides the date each agency first issued a credit risk rating to a given sovereign.

Table B1: Moody's Global Long-Term Rating Scale

| Rating | Description |
|---|---|
| Aaa | Obligations rated Aaa are judged to be the highest quality, subject to the lowest level of credit risk. |
| Aa | Obligations rated Aa are judged to be of high quality and are subject to very low credit risk. |
| A | Obligations rate A are judged to be upper-medium grade and are subject to low credit risk. |
| Baa | Obligations rated Baa are judged to be medium-grade and subject to moderate credit and as such may possess certain speculative characteristics. |
| Ba | Obligations rated Ba are judged to be speculative and are subject to substantial credit risk. |
| B | Obligations rated B are considered speculative and are subject to high credit risk. |
| Caa | Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk. |
| Ca | Obligations rated Ca are very highly speculative and are likely in, or very near, default with some prospect of principal and interest. |
| C | Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest. |

Note: Moody's appends numerical modifiers 1, 2, and 3 to each generic rating classification from Aa through Caa. The modifier 1 indicates that the obligation ranks in the higher end of its generic rating category, the modifier 2 indicates a mid-range ranking, and the modifier 3 indicates a ranking in the lower end of that generic rating category.

Table B2: Fitch International Credit Rating Scale

| Rating | Description |
|---|---|
| AAA | Highest credit quality. AAA ratings denote the lowest expectation of default risk. They are assigned only in cases of exceptionally strong capacity for payment of financial commitments. This capacity is highly unlikely to be adversely affected by foreseeable events. |
| AA | Very high credit quality. AA ratings denote expectations of very low default risk. They indicate very strong capacity for payment of financial commitments. This capacity is not significantly vulnerable to foreseeable events. |
| A | High credit quality. A ratings denote expectations of low default risk. The capacity for payment of financial commitments is considered strong. This capacity may, nevertheless, be more vulnerable to adverse business or economic conditions than is the case for higher ratings. |
| BBB | Good credit quality. BBB ratings indicate that expectations of default risk are currently low. The capacity for payment of financial commitments is considered adequate, but adverse business or economic conditions are more likely to impair this capacity. |
| BB | Speculative. BB ratings indicate an elevated vulnerability to default risk, particularly in the event of adverse changes in business or economic conditions over time; however, business or financial flexibility exists that supports the servicing of financial commitments. |
| B | Highly speculative. B ratings indicate that material default risk is present, but a limited margin of safety remains. Financial commitments are currently being met; however, capacity for continued payment is vulnerable to deterioration in the business and economic environment. |
| CCC | Substantial credit risk. Default is a real possibility. |
| CC | Very high levels of credit risk. Default of some kind appears probable. |
| C | Near default. A default or default-like process has begun, or the issuer is in standstill, or for a closed funding vehicle, payment capacity is irrevocably impaired. |
| RD | Restricted default. |
| D | D ratings indicate an issuer that in Fitch's opinion has entered into bankruptcy filings |

Note: Within rating categories, Fitch may use modifiers. The modifiers "+" or "-" may be appended to a rating to denote relative status within major rating categories. Such suffixes are not added to AAA ratings and ratings below the CCC category.

Unlike the III that is updated each semester, credit rating changes can occur at any time for an individual sovereign. In order to merge credit ratings data with the III, we use the credit rating that has been assigned the longest to a sovereign during a particular semester and merge that rating with the respective semester III reading. Since the III is a continuous variable and credit rating are a discrete variable (i.e. factor variable over the ordinal ratings labels), we visualize their correlation with box plots.

Table B3: Credit Agency Rating's First Issued Date

| Country | Moody's | Fitch |
| --- | --- | --- |
| Argentina | 11/18/1986 | 5/28/1997 |
| Brazil | 11/18/1986 | 12/1/1994 |
| China | 5/18/1988 | 12/11/1997 |
| Colombia | 8/4/1993 | 8/10/1994 |
| Ecuador | 7/24/1997 | 11/8/2002 |
| Egypt | 10/9/1996 | 8/19/1997 |
| Gabon | | 10/29/2007 |
| India | 1/28/1988 | 3/8/2000 |
| Iran | | 5/10/2002 |
| Iraq | | 8/7/2015 |
| Kazakstan | 11/11/1996 | 11/5/1996 |
| Kuwait | 1/29/1996 | 12/20/1995 |
| Malaysia | 1/18/1986 | 8/13/1998 |
| Mexico | 12/18/1990 | 8/30/1995 |
| Oman | 1/29/1996 | |
| Qatar | 1/29/1996 | 3/6/2015 |
| Russia | 10/7/1996 | 10/7/1996 |
| Saudi Arabia | 1/29/1996 | 11/24/2004 |
| Venezuela | 12/29/1976 | 9/15/1997 |

Box plots are used to show the overall dispersion of a continuous variable over groups. In our case, the y-axis is the continuous III, and the x-axis is the agency's credit rating ranks. The credit rating ranks are ordered along the x-axis from highest to lowest credit risk (from left to right). The box plots then graphs the quartiles of III observations over each credit risk rating. The horizontal line across the middle of the box is the median. The second quartile is the region from the median line to the bottom of the box, while the third quarter is the region from the median line to the top of the box. The bottom end of the lower whisker is the smallest value excluding outliers and the top end of the upper whisker is the largest value excluding outliers. Outliers are plotted as dots above and below the whisker of the box. Outliers above the upper whisker are 1.5 times greater than the third quartile while outliers below the lower whisker are 1.5 times lower than the first quartile. Figure B1 plots the III over Moody's credit risk ratings, and figure B2 plots the III over Fitch credit risk ratings.

Figure B1: Moody's Long-Term Sovereign Credit Ratings over the III

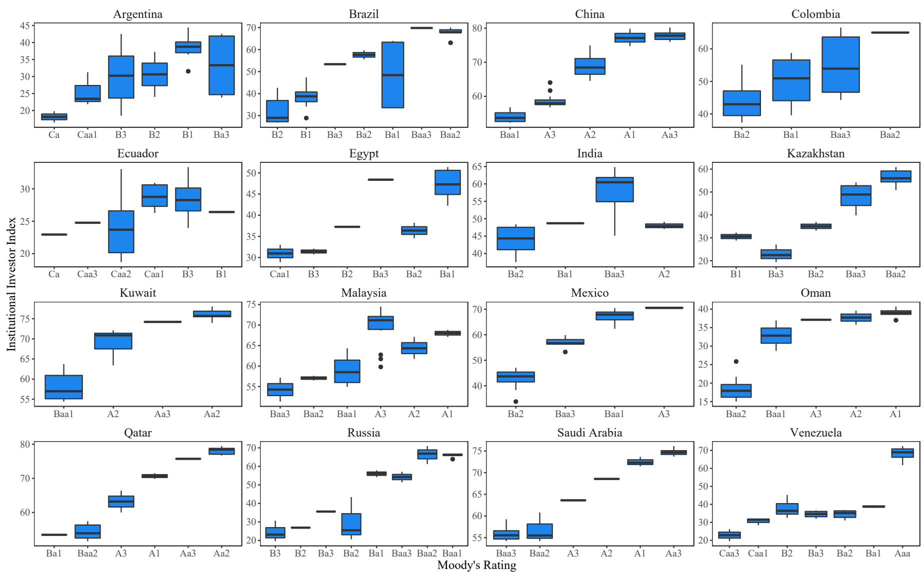
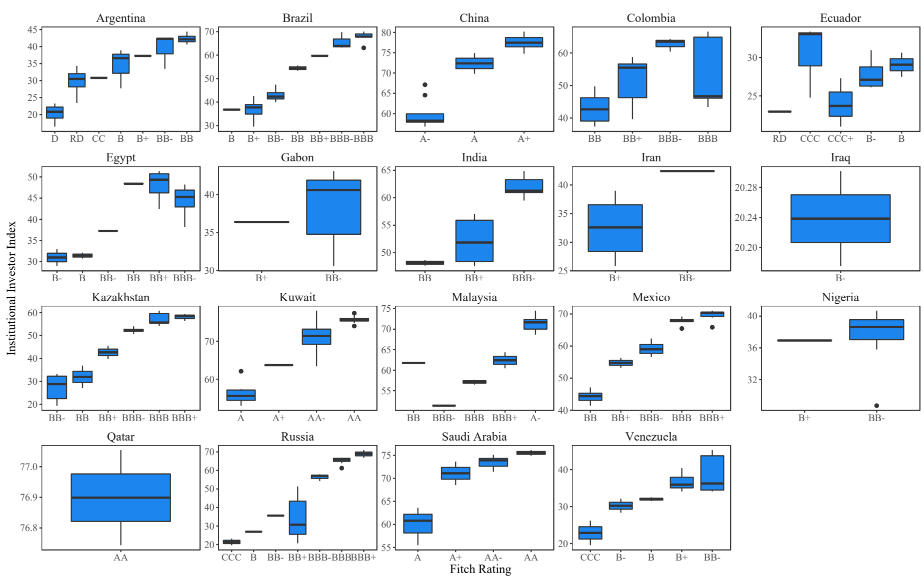



Figure B2: Fitch Long-Term Sovereign Credit Ratings over the III




We can see from the distributional characteristics of III over the Moody's and Fitch credit risk ratings that each sovereign's corresponding III measure tends to increase as its credit rating improves. This indicates that the III is correlated with credit ratings.

## B.2 Emerging Markets Bond Index (EMBI)

The Emerging Market Bond Index (EMBI) is JP Morgan's index of dollar denominated bonds issued for various emerging economies. It is one of the most widely used benchmarks of emerging market sovereign debt. The index comprises of US dollar-denominated Brady bonds, loans, and Eurobonds that have a face value of $500 million dollars or more and have a maturity greater than a year. The EMBI is quoted as a spread on sovereign debt over US treasuries, and the III is a measure of sovereign risk where 0 indicates high risk of default and 100 indicates low risk of default. Thus we expect to see these two move in opposite directions if the III is a good indicator of sovereign risk. In other words we expect the EMBI to rise as sovereign risk increases. Indeed we see in table B4 that the EMBI and III are negatively correlated, moving in the same direction to indicate sovereign risk.

Table B4: Correlation Between EMBI and III

| Country | Correlation |
|---|---|
| Angola | -0.570 |
| Argentina | -0.751 |
| Azerbaijan | 0.031 |
| Brazil | -0.789 |
| China | 0.312 |
| Colombia | -0.740 |
| Ecuador | -0.442 |
| Egypt | -0.642 |
| Gabon | -0.667 |
| India | -0.186 |
| Indonesia | -0.167 |
| Iraq | -0.163 |
| Kazakhstan | -0.293 |
| Malaysia | -0.434 |
| Mexico | -0.723 |
| Nigeria | -0.666 |
| Russian Federation | -0.686 |
| Venezuela | -0.629 |
| Vietnam | 0.146 |

Since the EMBI was introduced only in 1992, we have fewer observations of the EMBI than we have for the III. Following Erb et al. (1996), we can use the fact that the EMBI and the III are correlated with each other to extend the EMBI backwards so that it starts in the same year as the III for country i.

We use the following equation to build the index for each country:

$$EMBI_t = \alpha_0 + \alpha_1 III_t + \epsilon_t \tag{B1}$$

Suppose we have observations of the $EMBI$ for country $i$ starting at time $t$ through $T$ where $t < T$. We estimate (B1) using observations $t$ through $T$ of the $EMBI$ and $III$ for country $i$. Table B5 reports the estimates for $\alpha_1$ in (B1) for each country. We see that most country's estimate is negative and statistically significant. This implies that equation (B1) is an appropriate model to use to estimate values of the EMBI that are not available. We are then able to plug observation $III_{t-1}$ into the estimated model to calculate the fitted value for $EMBI_{t-1}$. Now we re-estimate (B1) using observations $t-1$ through $T$ of the $EMBI$ and $III$, and then plug observation $III_{t-2}$ into the newly estimated model to calculate the fitted value for $EMBI_{t-2}$. We continue this back-substitution until we have exhausted all observations of the III for country i. Our final output is an index of the EMBI re-constructed to the same time as the first observation of the III for country i. Figures of our reconstructed EMBI indices are available upon request.

Table B5: $\alpha_1$ Estimates on Observed Values of the EMBI

| Country | Slope Coefficient | Standard Error |
|---|---|---|
| Angola | $-61.18$ | 35.98 |
| Argentina | $-156.10^{***}$ | 20.91 |
| Azerbaijan | $1.44$ | 17.76 |
| Brazil | $-21.82^{***}$ | 2.59 |
| China | $1.75^{**}$ | 0.81 |
| Colombia | $-14.98^{***}$ | 2.24 |
| Ecuador | $-84.03^{***}$ | 26.61 |
| Egypt | $-14.14^{***}$ | 3.19 |
| Gabon | $-37.46^{***}$ | 10.79 |
| India | $-6.00$ | 12.96 |
| Indonesia | $-2.41$ | 2.95 |
| Iraq | $-5.01$ | 6.96 |
| Kazakhstan | $-18.56$ | 15.15 |
| Malaysia | $-7.38^{***}$ | 2.49 |
| Mexico | $-17.11^{***}$ | 2.49 |
| Nigeria | $-42.23^{***}$ | 7.88 |
| Russian Federation | $-41.69^{***}$ | 7.47 |
| Venezuela | $-74.32^{***}$ | 13.99 |
| Vietnam | $6.89$ | 10.43 |

*** p<0.01, ** p<0.05, * p<0.1

## C III, Oil Production and External Debt

Figure C1 plots the relationship between the III and oil production value to GDP ratio, for each country, over the period 1979-2010. One feature stands out from Figure C1: when oil production value to GDP ratio is high, the country risk index tends to improve. Note that there are countries where the correlation is not significant, such as Iran, United Kingdom, Egypt or Gabon.

Figure C1: Institutional Investor Index (X-Axis) and oil production value to GDP (%, Y-Axis).

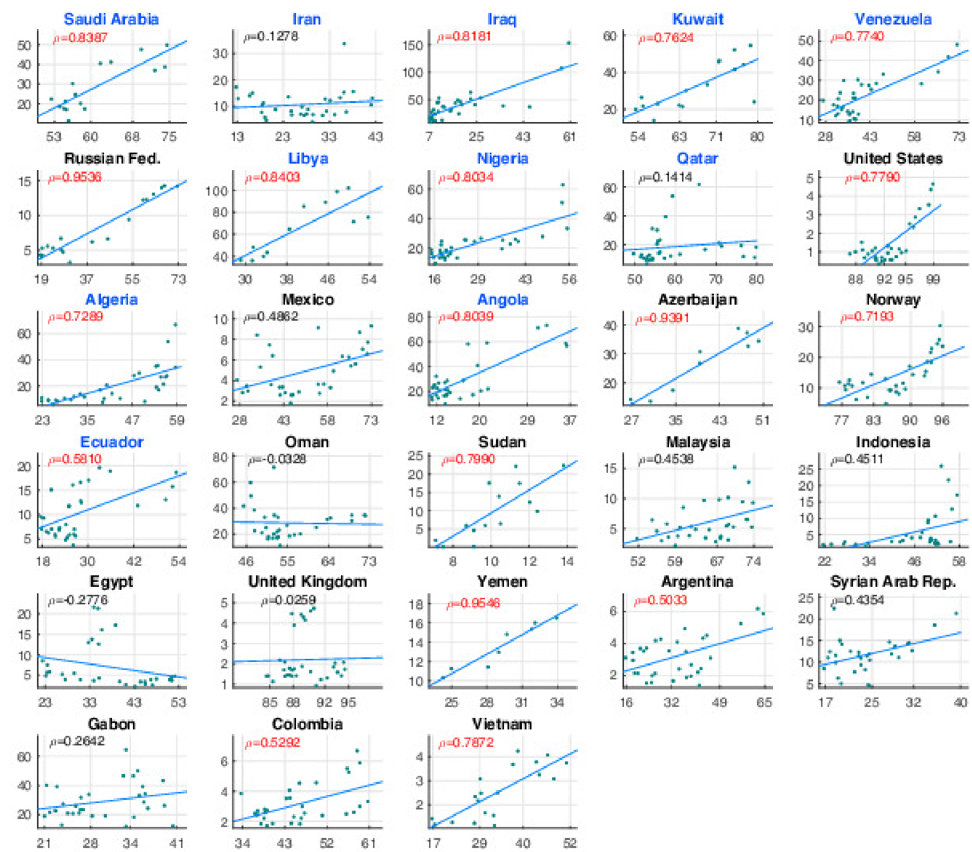
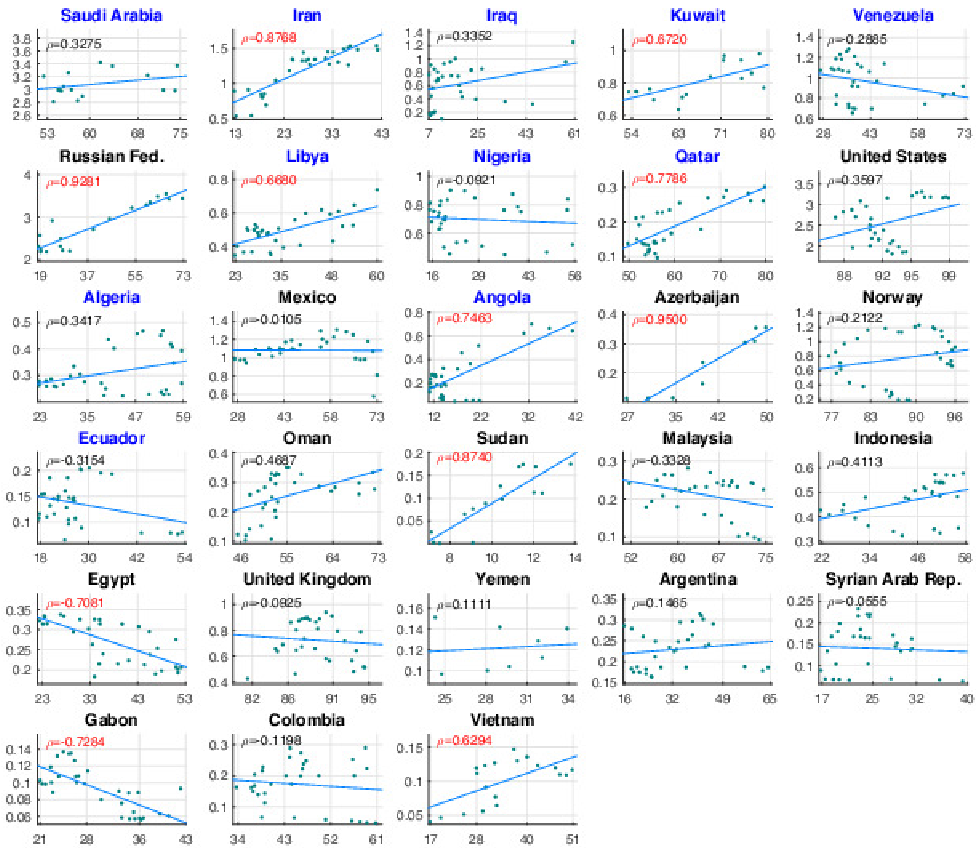


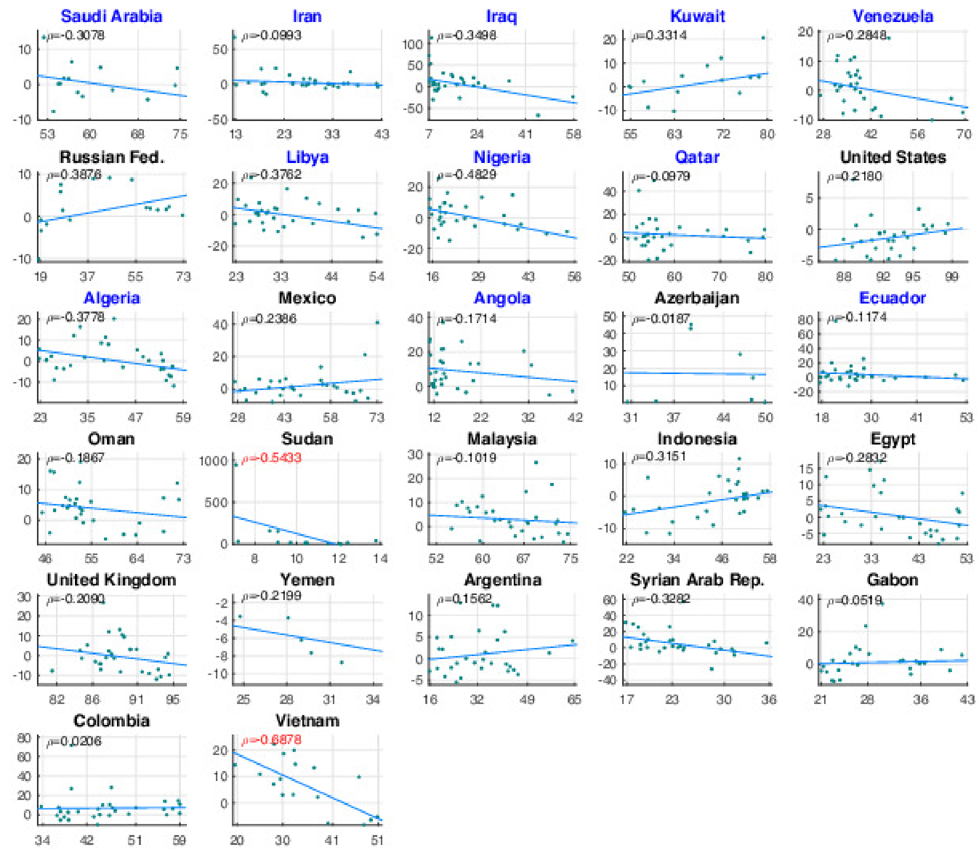

Figure C2 presents the III versus oil production (in billion barrels per year).

Figure C2: Institutional Investor Index (X-Axis) and oil production (billion barrels per year, Y-Axis).



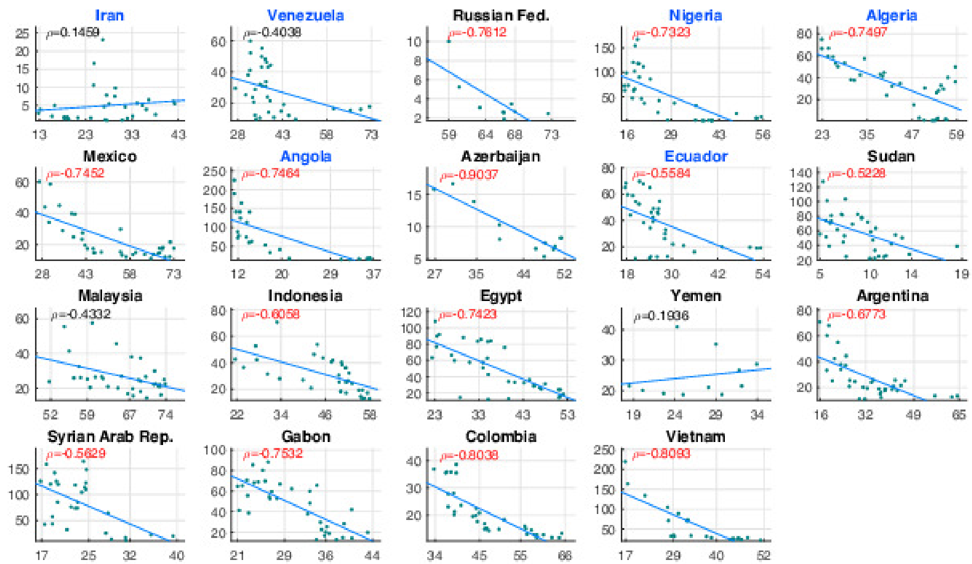
In this figure, absolute value of correlation coefficients greater than 0.5 are displayed in red. As we can see, there is not a clear pattern, since there are some countries for which the relationship is clearly positive, while for others it is negative or zero. This suggests that oil price is the "main driving force" behind changes in the country risk index (and not oil production). In Figure C3 we document the association between III and the oil production growth rate.

Figure C3: Institutional Investor Index (X-Axis) and oil production growth rate (%, Y-Axis).



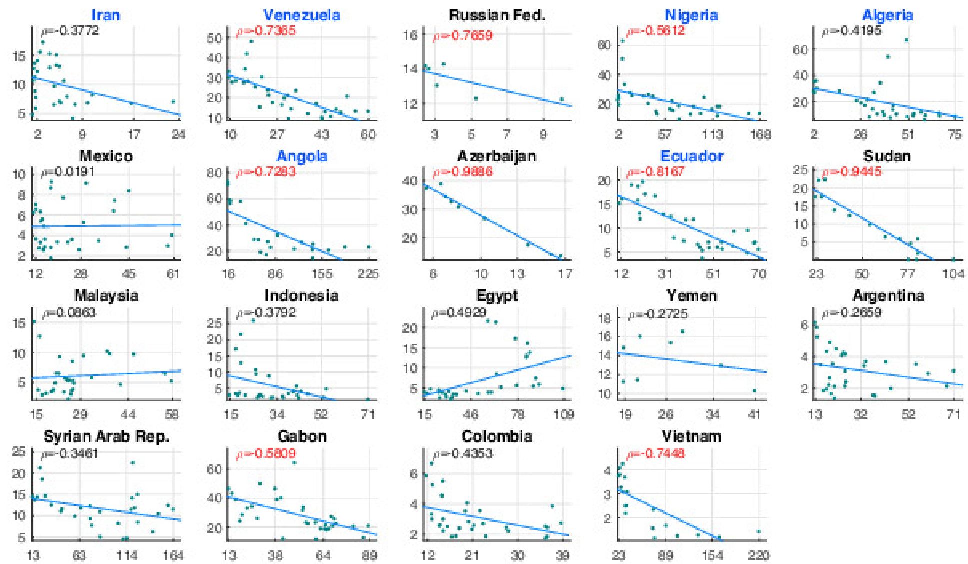

In this case, correlation coefficients lower than -0.5 are displayed in red. The results point in the direction that there is not any association between these two variables, although a negative relationship is observed for Sudan and Vietnam. Additionally, Figure C4 shows the relationship between the III and total public external debt to GDP ratio.

Figure C4: Institutional Investor Index (X-Axis) and total external public debt to GDP (%, Y-Axis).




Note that for most countries, correlation coefficients are displayed in red, which means that these are lower than -0.5. As we can see, III goes down when total public external debt increases. Additionally, Figure C5 shows the association between total external public debt to GDP ratio and oil production value to GDP ratio.

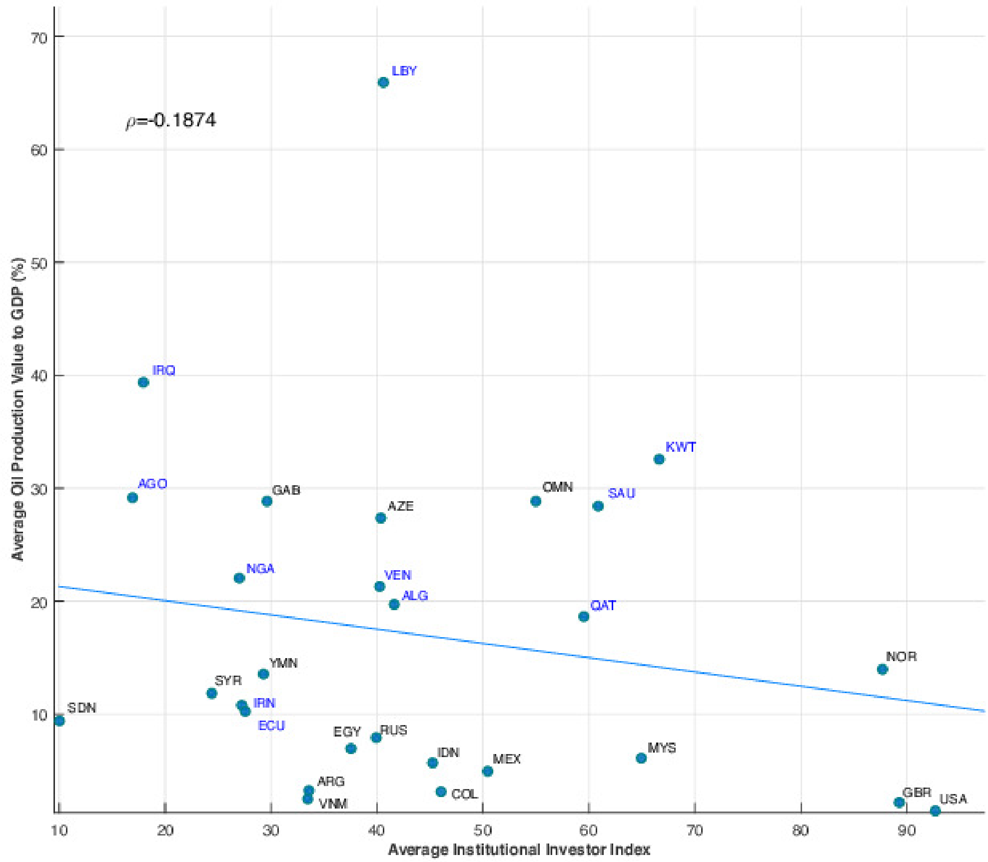
Figure C5: Total external public debt to GDP (%, X-Axis) and oil production value to GDP (%, Y-Axis).




As we can see, for 9 countries there is a negative correlation, which implies than when oil production value to GDP is high, total public external debt tends to be low. Nevertheless, such a contention is not reinforced by the rest of countries in the sample, since no significance is observed. Moreover, in the case of Egypt, the estimated coefficient shows almost a positive and statistically strong effect.

Moreover, Figure C6 plots the average III against average oil production value to GDP: In this case, we compute a low correlation coefficient (-0.187). The negative trend indicates that countries with high oil production value to GDP over time show a high country risk (or a low average III). It is important to mention that average oil production value to GDP may be low because historical GDP is very high when compared with the historical oil production value, such as in USA or Norway. Furthermore, this negative relationship may also be driven by exceptional cases such as Libya or Iraq, which have average oil production value to GDP of about 67 and 39 percent, and average III of about 41 and 17, respectively.

Figure C6: Average Institutional Investor Index (X-Axis) and average oil production value to GDP (Y-Axis): 1979-2010.



# D Panel Estimation Approach

Before proceeding to dynamic panel data models, we need to verify that all variables are integrated of the same order. In doing so, we have used the test of the panel unit root of Im et al. (2003) IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF henceforth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregressive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{D1}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where $N$ refers to the number of countries in the panel and $T$ refers to the number of observations over time. In this case, $y_i$ stands for each variable under consideration in our model (for example, III, oil GDP or non-oil GDP), $\alpha_i$ is the individual fixed effect and $p$ is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all $i$, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where $N_1$ denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{D2}$$

where $t_i^{ADF}$ is the ADF t-statistic for country $i$, taking into account the country specific ADF regression, given by (D1). The $\bar{t}$ statistic has been shown to be normally distributed under $H_0$. Table D1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both $T$ and $N$ are large. At one extreme, the usual practice is either to estimate $N$ separate regressions

Table D1: Im et al. (2003) panel unit root test outcome: 1979-2010

| | Levels | | Logs | |
|---|---|---|---|---|
| | $t$-statistic | $P$-value | $t$-statistic | $P$-value |
| Inst. Inv. | 0.280 | 0.610 | 0.293 | 0.615 |
| Δ Inst. Inv. | -11.629 | 0.000 | -11.645 | 0.000 |
| Oil GDP | 5.286 | 1.000 | 0.680 | 0.752 |
| Δ Oil GDP | -11.972 | 0.000 | -13.776 | 0.000 |
| Non-oil GDP | 14.801 | 1.000 | 2.247 | 0.988 |
| Δ Non-oil GDP | -7.413 | 0.000 | -10.345 | 0.000 |
| Oil Reserves | 4.376 | 1.000 | 2.404 | 0.992 |
| Δ Oil Reserves | -13.954 | 0.000 | -14.352 | 0.000 |
| Ext. pub. debt to GDP | 1.113 | 0.867 | 3.727 | 1.000 |
| Δ Ext. pub. debt to GDP | -12.196 | 0.000 | -11.045 | 0.000 |
| NFA | 0.117 | 0.546 | . | . |
| Δ NFA | -9.364 | 0.000 | . | . |

Note: When computing NFA outcome, we excluded Iraq because of data limitations.

and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran & Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error variances are constrained to be the same. In this case, the model controls for all time-invariant differences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran et al. (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i}+\theta_{1i}OilGDP_{it}+\theta_{2i}NonOilGDP_{it}+\theta_{3i}OilR_{it}+\theta_{4i}X_{it}+\theta_{5i}Default_{it}+\mu_i+\epsilon_{it} \quad \text{(D3)}$$

Again, each observation is subscripted for the country $i$ and the year $t$. In this case, $X \in \{ExtPubD, OilDisc, NFA\}$. The variable $III$ is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $ExtPubD$ is the external public debt to GDP ratio, $OilDisc$ is the log of oil discoveries, $NFA$ corresponds to net foreign assets to GDP ratio, and $Default$ is a dummy variable that the country is in default. Additionally, $\mu_i$ is a set of country fixed effects (such as geographical or institutional factors) and $\epsilon_{it}$ is the idiosyncratic error term.

Now, with a maximum lag of one for all variables except $Default$, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1,1,0) dynamic panel specification of (D3):

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i}OilGDP_{it} + \delta_{11i}OilGDP_{i,t-1} + \delta_{20i}NonOilGDP_{it}+$$
$$\delta_{21i}NonOilGDP_{i,t-1}+\delta_{30i}OilR_{it}+\delta_{31i}OilR_{i,t-1}+\delta_{40i}X_{it}+\delta_{41i}X_{i,t-1}+\theta_{5i}Default_{it}+\mu_i+\epsilon_{it} \quad \text{(D4)}$$

Then, the error correction equation of (D4) is:

$$\Delta III_{it} = \phi_i \Big( III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} - \hat{\theta}_{5i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it} \quad \text{(D5)}$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i}$$
$$\hat{\theta_{3i}} = \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i}; \hat{\theta_{5i}} = \frac{\theta_{5i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, $\phi_i$ is the error correction speed of adjustment parameter, and we would expect $\phi_i$ to be negative if the variables exhibit a return to long-run equilibrium[1].

[1] Replacing $\hat{\theta}_i$-parameters and $\phi_i$ in equation (D3) we get:

$$\Delta III_{it} = -(1-\lambda_i)\Big( III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i} OilR_{it} - \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i} X_{it} - \frac{\theta_{5i}}{1-\lambda_i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Removing similar terms, the above expression is as follows:

$$\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} + (\delta_{40i}+\delta_{41i}) X_{it} + \theta_{5i} Default_{it} - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Rewriting:

$$\begin{aligned} III_{it} - III_{i,t-1} = &- (1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} \\ &+ (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}(OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i}(NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ &- \delta_{31i}(OilR_{it} - OilR_{i,t-1}) - \delta_{41i}(X_{it} - X_{i,t-1}) + \theta_{5i} Default_{it} + \epsilon_{it} \end{aligned}$$

Again, simplifying this equality we obtain:

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \theta_{5i} Default_{it} + \mu_i + \epsilon_{it}$$

Note that this expression is equivalent to (D4). For a long-run relationship to exist, we require that $\phi \neq 0$.

Table D2: Hausman test outcome: 1979-2010

| | Model (1) | | Model (2) | | Model (3) | |
|---|---|---|---|---|---|---|
| | $\chi^2$-stat | P-value | $\chi^2$-stat | P-value | $\chi^2$-stat | P-value |
| MG vs. DFE | 0.02 | 1.000 | 0.01 | 1.000 | 0.06 | 1.000 |
| PMG vs. DFE | 0.03 | 1.000 | 0.03 | 1.000 | 0.03 | 1.000 |
| MG vs. PMG | 4.42 | 0.491 | 5.05 | 0.537 | 8.99 | 0.174 |

### D.1 Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (D5). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for China, India, and Brazil since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = (\beta_{DFE} - \beta_{PMG})' \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} (\beta_{DFE} - \beta_{PMG}) \sim \chi^2$$

where $\beta_j$ is the vector of coefficients and var$(\beta_j)$ is the covariance matrix of $\beta_j$, estimated using the $j-$technique, for $j$ =DFE, PMG. Under the null hypothesis, $H$ has asymptotically the $\chi^2$ distribution. Table D2 reports the results of Hausman test, in which Model (1) corresponds to equation (D5), excluding $NFA$ from $X_i$, while Model (2) excludes $Default$. Model (3) includes all variables in $X_i$ into the regressors.

Under the current specification, the hypothesis that the country risk equation (equation (D5)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in table D2 suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 1, which indicates that DFE is more efficient estimator than MG and

PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are 1. Because of this, we choose to employ the DFE estimator.

## E Oil Price Upswings and Downswings

Table E1: Oil Price Upswings and Downswings

| Downswings | | Upswings | |
|---|---|---|---|
| Period | Number of Months | Period | Number of Months |
| NOV 75 - OCT 78 | 36 | NOV 78 - JAN 81 | 27 |
| FEB 81 - JUL 86 | 66 | AUG 86 - JUL 87 | 12 |
| AUG 87 - NOV 88 | 16 | DEC 88 - OCT 90 | 23 |
| NOV 90 - DEC 93 | 38 | JAN 94 - OCT 96 | 34 |
| NOV 96 - DEC 98 | 26 | JAN 99 - SEP 00 | 21 |
| OCT 00 - DEC 01 | 15 | JAN 02 - JUL 08 | 79 |
| AUG 08 - MAY 10 | 22 | | |
| TOTAL | 219 | TOTAL | 196 |

## F Are all Oil Exporting Countries Price Takers?

This appendix examines whether the countries in our sample are price takers in the world market of oil.[2] We examine causality between a country's extraction and oil prices using two strategies, both in a bivariate context. First, we test on the levels, using a modified version of the Granger causality test proposed by Toda & Yamamoto (1995). Second, we test causality using the Granger test on the first differences of both series.

For the causality test a modified Wald test (MWALD) is used as proposed by Toda & Yamamoto (1995) that avoids the problems associated with the ordinary Granger causality test by ignoring any possible non-stationary or cointegration between series when testing for causality. [3]The Toda & Yamamoto (1995) approach fits a standard vector autoregressive model in the levels of the variables (rather than the first differences, as the case with Granger causality tests) thereby minimizing the risks associated with the possibility of wrongly identifying the order of integration of the series.

The basic idea of this approach is to artificially augment our bivariate VAR or order k, by the maximal order of integration, one in this case. Once this is done, a (k+1)-th order of VAR is estimated and the coefficients of the last one lagged vector is ignored. The application of the Toda & Yamamoto (1995) procedure ensures that the usual Wald test statistic for Granger causality has the standard asymptotic distribution hence valid inference can be done.

Lag length for VAR are chosen based on information criteria (Akaike, Schwarz and Hanna-Quinn), when there is not agreement between those indicators, pormanteau (bivariate Lung-Box statistic) test is used to decide. This statistics joint with its P-values are contained and third and four columns of tables F1 and F2.

[2]We are grateful to Norberto Rodriguez-NiÒo from the Banco de la República de Colombia for his assistance with this analysis.

[3]As quoted from Wolde-Rufael (2005) "... given that unit root and cointegration tests have low power against the alternative, these tests can be misplaced and can suffer from pre-testing bias (see Pesaran et al. (2001); Toda & Yamamoto (1995)). Moreover, as demonstrated by Toda & Yamamoto (1995), the conventional F-statistic used to test for Granger causality may not be valid as the test does not have a standard distribution when the time series data are integrated or cointegrated."

### F.1 Data

We used monthly data of crude oil for the 20 major exporting countries; the sample period cover from January 2002 to November 2016. The data source is Joint Oil Data Initiative (JODI) Database (available at http://www.jodidb.org/TableViewer/tableView.aspx). For Colombia, the figures have source Banco de la Rep˙blica and are based on DIAN-DANE. Units are thousand barrels per period. Exports the top 20 countries accounted for approximately 96% of reported crude oil exports at the JODI base in 2015.

### F.2 Results

Unit root test results (not presented here but available up to request) show that all the variables are integrated of order one.

Table F1 shows the results for the TY test. It is worth to remain that the null hypothesis in this as next table is that of non-causality. Table F2 presents results for Granger causality test, for the series in differences. Results in both tables coincide signaling oil exports from United Arab Emirates, Oman, Brazil and Azerbaijan causing (in Granger sense) oil prices. TY shows that exports from Canada also G-cause prices, and model in differences indicated that Kuwait G-cause oil prices.

Table F1: Taro-Yamamoto test results for series in levels

| Country | Lag | Lung-Box | | Jarque-Bera | | Taro-Yamamoto | | |
|---|---|---|---|---|---|---|---|---|
| | | Q-Stat | P-Value | Stat | P-Value | Statistic | P-Value | Decision |
| Saudi Arabia | 2 | 26.75 | 0.32 | 164.12 | 0.00 | 1.47 | 0.48 | |
| Russia | 2 | 30.14 | 0.18 | 75.23 | 0.00 | 1.90 | 0.39 | |
| Iraq | 2 | 28.48 | 0.24 | 50.22 | 0.00 | 1.28 | 0.53 | |
| U. Arab Emir. | 2 | 29.43 | 0.20 | 31.25 | 0.00 | 17.32 | 0.00 | Cause |
| Canada | 2 | 26.31 | 0.34 | 70.50 | 0.00 | 7.30 | 0.03 | Cause |
| Nigeria | 2 | 17.33 | 0.83 | 13.42 | 0.01 | 0.99 | 0.61 | |
| Kuwait | 2 | 21.64 | 0.60 | 23.36 | 0.00 | 1.20 | 0.55 | |
| Angola | 4 | 23.88 | 0.09 | 17.30 | 0.00 | 7.86 | 0.10 | |
| Venezuela | 2 | 23.61 | 0.48 | 46.25 | 0.00 | 5.17 | 0.08 | |
| Iran | 2 | 27.83 | 0.27 | 66.94 | 0.00 | 5.00 | 0.08 | |
| Mexico | 2 | 21.95 | 0.58 | 14.50 | 0.01 | 4.19 | 0.12 | |
| Norway | 3 | 18.43 | 0.56 | 6.47 | 0.17 | 3.45 | 0.33 | |
| Oman | 2 | 18.92 | 0.76 | 4320.42 | 0.00 | 9.10 | 0.01 | Cause |
| Brasil | 7 | 3.17 | 0.53 | 24.80 | 0.00 | 16.69 | 0.02 | Cause |
| Azerbaijan | 2 | 20.98 | 0.64 | 1171.78 | 0.00 | 13.11 | 0.00 | Cause |
| Uni. Kingdom | 2 | 28.88 | 0.22 | 22.93 | 0.00 | 0.10 | 0.95 | |
| Algeria | 2 | 29.15 | 0.21 | 12.62 | 0.01 | 4.89 | 0.09 | |
| Qatar | 2 | 20.04 | 0.69 | 127.50 | 0.00 | 1.84 | 0.40 | |
| USA | 3 | 21.69 | 0.36 | 332.23 | 0.00 | 1.19 | 0.76 | |
| Colombia | 3 | 25.90 | 0.17 | 13.44 | 0.01 | 0.81 | 0.85 | |

Table F2: Granger tets results for series in diferences

| Country | Lag | Lung-Box | | Jarque-Bera | | Taro-Yamamoto | | |
|---|---|---|---|---|---|---|---|---|
| | | Q-Stat | P-Value | Stat | P-Value | Statistic | P-Value | Decision |
| Saudi Arabia | 7 | 6.69 | 0.15 | 49.06 | 0.00 | 10.77 | 0.15 | |
| Russia | 6 | 14.45 | 0.07 | 132.17 | 0.00 | 7.56 | 0.27 | |
| Iraq | 2 | 34.37 | 0.08 | 6.76 | 0.15 | 3.41 | 0.18 | |
| U. Arab Emir | 6 | 9.38 | 0.31 | 71.06 | 0.00 | 18.78 | 0.00 | Cause |
| Canada | 6 | 12.82 | 0.12 | 5.86 | 0.21 | 7.58 | 0.27 | |
| Nigeria | 1 | 37.67 | 0.10 | 25.24 | 0.00 | 0.33 | 0.57 | |
| Kuwait | 6 | 6.57 | 0.58 | 14.00 | 0.01 | 13.63 | 0.03 | Cause |
| Angola | 6 | 8.01 | 0.43 | 342.62 | 0.00 | 10.84 | 0.09 | |
| Venezuela | 1 | 26.57 | 0.54 | 16.27 | 0.00 | 2.14 | 0.14 | |
| Iran | 2 | 34.39 | 0.08 | 95.29 | 0.00 | 2.96 | 0.23 | |
| Mexico | 2 | 28.31 | 0.25 | 32.99 | 0.00 | 2.65 | 0.27 | |
| Norway | 2 | 32.64 | 0.11 | 20.19 | 0.00 | 3.26 | 0.20 | |
| Oman | 6 | 10.13 | 0.26 | 13053.21 | 0.00 | 26.42 | 0.00 | Cause |
| Brazil | 7 | 8.94 | 0.06 | 265.77 | 0.00 | 18.39 | 0.01 | Cause |
| Azerbaijan | 2 | 32.15 | 0.12 | 1029.34 | 0.00 | 12.68 | 0.00 | Cause |
| Uni. Kingdom | 6 | 14.49 | 0.07 | 27.07 | 0.00 | 5.27 | 0.51 | |
| Algeria | 2 | 33.76 | 0.09 | 7.44 | 0.11 | 3.82 | 0.15 | |
| Qatar | 6 | 7.20 | 0.51 | 87.55 | 0.00 | 12.24 | 0.06 | |
| USA | 3 | 35.85 | 0.02 | 33.07 | 0.00 | 2.56 | 0.46 | |
| Colombia | 2 | 29.64 | 0.20 | 18.90 | 0.00 | 1.43 | 0.49 | |

## G Model Variants under Commitment

We analyze here three variants of the model under the assumption that the planner is committed to repay. The planner's optimization problem is characterized in a generic form that allows us to capture cases in which the planner accesses world financial markets facing with either a given bond pricing function that depends on the planner's debt and reserves) or a constant world real interest rate, and a case in which the planner operates under financial autarky. The latter coincides with the solution of the default payoff if default triggers permanent exclusion from credit markets.

The generic planner's problem in sequential form is the following:

$$\max_{c_t, x_t, b_{t+1}, s_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{G1}$$

s.t.

$$c_t + e\left(x_t, s_t\right) = y_t + p_t x_t - q\left(s_{t+1}, b_{t+1}\right) b_{t+1} + b_t \tag{G2}$$

$$s_{t+1} = s_t - x_t + \kappa \tag{G3}$$

$$x_t \geq 0 \tag{G4}$$

$$x_t \leq s_t + \kappa. \tag{G5}$$

The first constraint is the resource constraint, where $q\left(s_{t+1}, b_{t+1}\right)$ is an ad-hoc pricing function of bonds that is assumed to be the equilibrium pricing function of the model with default and satisfies the following assumptions: $q\left(\cdot\right)$ is continuously differentiable, strictly concave and increasing in $b_{t+1}$ for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given $s_{t+1}$ (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 1), with $q\left(\cdot\right) = q^*$ for $b_{t+1} \geq 0$ and $q\left(\cdot\right) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q\left(\cdot\right)$ is also increasing and concave in $s_{t+1}$ for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$, $s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}$ is the minimum $s_{t+1}$ needed for profits to be non-negative, and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given $b_{t+1}$ (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in $s_{t+1}$ and $\overline{s}(b_{t+1})$ is increasing in $b_{t+1}$. In addition, we assume shocks are i.i.d so that $q(\cdot)$ is independent of $p_t$ and $y_t$. The second constraint is the law of motion of reserves. The third and fourth constraints are the feasibility boundaries of oil extraction.

The first-order conditions are:

$$\lambda_t = u'(c_t) \tag{G6}$$

$$\lambda_t \left[p_t - e_x(x_t, s_t)\right] + \psi_t^l = \mu_t + \psi_t^u \tag{G7}$$

$$\begin{aligned} u'(c_t)\left[p_t - e_x(x_t, s_t) + q_s(s_{t+1}, b_{t+1})\, b_{t+1}\right] + \psi_t^l - \psi_t^u = \\ \beta E_t\left[u'(c_{t+1})\left(p_{t+1} - e_x(x_{t+1}, s_{t+1}) - e_s(x_{t+1}, s_{t+1})\right) + \psi_{t+1}^l\right] \end{aligned} \tag{G8}$$

$$u'(c_t)\left[q(s_{t+1}, b_{t+1}) + q_b(s_{t+1}, b_{t+1})\, b_{t+1}\right] = \beta E_t\left[u'(c_{t+1})\right]. \tag{G9}$$

where $\lambda_t$ is multiplier on the resource constraint, $\mu_t$ is the multiplier on the law of motion of reserves, and $\psi_t^h$ and $\psi_t^l$ are the multipliers on the upper and lower feasibility constraints on oil extraction.

Defining the planner's return on bonds as $R^b(s_{t+1}, b_{t+1}) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$, which is decreasing in $b_{t+1}$ (i.e. the planner's real interest rate increases with debt) because of the assumed properties of $q(\cdot)$, the Euler equation for bonds (eq (G9)) implies:[4]

$$u'(c_t) = R^b(s_{t+1}, b_{t+1})\, \beta E_t\left[u'(c_{t+1})\right]. \tag{G10}$$

Notice that, as implied by the definition of $R^b$, in evaluating the marginal benefit of borrowing in the right-hand-side of this expression, the planner internalizes that borrowing more (i.e. making $b_{t+1}$ "more negative") increases the cost of borrowing.

The rate of return on oil extraction is defined as $R_{t+1}^O \equiv \frac{q_{t+1}^O + d_{t+1}^O}{q_t^O}$, where $q_t^O$ is the asset price of oil defined as $q_t^O \equiv p_t - e_x(t) + \Delta\tilde{\psi}_t$ (where $\Delta\tilde{\psi}_t \equiv \tilde{\psi}_{t+1}^l - \tilde{\psi}_{t+1}^h$ and $\tilde{\psi}_t^i = \psi_t^i / u'(t)$ for $i = h, l$) and $d_{t+1}^O$ is the dividend from oil extraction at $t+1$ defined as $d_{t+1}^O \equiv -e_s(t+1) + \tilde{\psi}_{t+1}^h$. Notice that $d_{t+1}^O > 0$ because $e_s(t+1) < 0$ and $\tilde{\psi}_{t+1}^h \geq 0$. The Euler equation for oil reserves (eq. (G8)) can then be rewritten as:

$$u'(c_t)\left[1 + \frac{q_s(s_{t+1}, b_{t+1})\, b_{t+1}}{q_t^O}\right] = \beta E_t\left[u'(c_{t+1})\, R_{t+1}^O\right]. \tag{G11}$$

The left-hand-side of this expression shows that in evaluating the marginal cost of accumulating additional reserves, the planner internalizes the fact that higher $s_{t+1}$ increases the price of bonds, so that if it plans to issue debt ($b_{t+1} < 0$), the higher price at which it can

[4]The derivative of $R^b(\cdot)$ w.r.t. $b_{t+1}$ is $R_b^b(\cdot) = \frac{-\left(2q_b(\cdot)+q_{bb}(\cdot)b_{t+1}\right)}{\left(q(\cdot)+q_b(\cdot)b_{t+1}\right)^2}$, and the properties that $q(s_{t+1}, b_{t+1}) = q^*$ for $b_{t+1} \geq 0$ and $q(s_{t+1}, b_{t+1})$ is strictly concave and increasing in $b_{t+1}$ for $b_{t+1} \in [-\bar{b}(s_{t+1}), 0]$ imply that $-\left(2q_b(\cdot) + q_{bb}(\cdot)b_{t+1}\right) > 0$ and hence $R_b^b(\cdot) < 0$ in that same interval.

be sold reduces the marginal cost of building reserves. Hence, we can also express the Euler equation of reserves redefining the rate of return on oil to impute this extra gain:

$$u'(c_t) = \beta E_t \left[ u'(c_{t+1}) \tilde{R}^O_{t+1} \right], \tag{G12}$$

where $\tilde{R}^O_{t+1} \equiv \frac{q^O_{t+1} + d^O_{t+1}}{\left[q^O_t + q_s(s_{t+1}, b_{t+1}) b_{t+1}\right]}$ is the rate of return on oil inclusive of the benefit of higher reserves increasing the price at which newly-issued debt is sold.

The above Euler equation can be used to solve forward for the asset price of oil. To this end, rewrite the equation as follows:

$$q^O_t + z_t = E_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left( q^O_{t+1} + d^O_{t+1} \right) \right] \tag{G13}$$

where $z_t \equiv q_s(t)\, b_{t+1}$ and $q_s(t)$ is the derivative with respect to reserves of the price of bonds sold at date $t$, which is a function of $(b_{t+1}, s_{t+1})$. Notice $z_t \leq 0$ because $q_s(\cdot) > 0$ for $b_{t+1} < 0$ and otherwise $q_s(\cdot) = 0$. Adding and subtracting $z_{t+1}$ to $q^O_{t+1}$ in the right-hand-side of this equation and solving forward yields:

$$q^O_t + z_t = E_t \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} [d^O_s - z_s] \right] > 0 \tag{G14}$$

The expression in the right-hand-side is positive because marginal utility is positive, $d^0_s > 0$ and $z_s \leq 0$. It follows then that $q^O_t + z_t > 0$, and since $z_s \leq 0$ we obtain $q^O_t > -z_t \geq 0$. Thus, the asset price of oil equals the expected present discounted value (discounted with the planner's stochastic discount factors) of the revenue stream composed of oil dividends plus the marginal revenue of selling bonds at a higher price when reserves increase. Or, the asset price of oil with this marginal revenue imputed, $\tilde{q}^0_t$ equals the expected present discounted value of the stream of oil dividends with the stream of these marginal revenues included $\tilde{q}^O_t = E_t \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} \tilde{d}^O_s \right]$, where $\tilde{d}^O_s \equiv d^O_s - z_s$.

Combining the Euler equations for bonds and reserves yields the following expression for the excess return on oil (the oil risk premium):

$$E_t \left[ R^o_{t+1} \right] - R^b_{t+1}(s_{t+1}, b_{t+1}) \left[ 1 + \frac{q_s(t+1) b_{t+1}}{q^O_t} \right] = -\frac{\text{cov}_t \left( u'(c_{t+1}), R^o_{t+1} \right)}{E_t \left[ u'(c_{t+1}) \right]}. \tag{G15}$$

The left-hand-side is the excess return relative to the yield on bonds inclusive of the effect of higher reserves on the resources generated by borrowing. Defined in this way, the excess return takes the standard form of an equity premium determined by the covariance of the planner's marginal utility and the rate of return on oil. Defining the return on oil with the effect of higher reserves increasing bond prices imputed, the excess return is:

$$E_t\left[\tilde{R}^o_{t+1}\right] - R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{G16}$$

We explore next three cases of this generic setup. First, a case in which the economy is in permanent financial autarky but can export oil. Second, a small-open-economy case in which the economy has access to a world credit market at a constant, exogenous price of bonds $q^*$, which is akin to an RBC model with oil extraction. Third, a case in which the planner faces the exogenous bond pricing function $q(b_{t+1}, s_{t+1})$. In each instance we discuss results with and without uncertainty.

## G.1 Financial Autarky

Consider first the case in which the economy is in financial autarky and there is no uncertainty. The Euler equation of reserves implies:

$$\frac{q^o_{t+1} + d^o_{t+1}}{q^o_t} = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)}. \tag{G17}$$

In turn, solving forward this condition yields a standard asset-pricing condition by which the asset price of oil equals the present discounted value of oil dividends discounted with the intertemporal discount factors:

$$q^O_t = \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} d^O_s \tag{G18}$$

Note that since $d^0_s > 0$ and $u'(s), u'(t) > 0$, it follows that $q^O_t > 0$.

In this case, the optimal extraction and reserves plans equate $R^o_t$ with the endogenous domestic real interest rate represented by the intertemporal marginal rate of substitution, each represented by the left- and right-hand-side of the reserves Euler equation, respectively. Oil extraction and reserves are used to smooth consumption.

The deterministic steady state is characterized by these two conditions:

$$\beta\left(q^{Oss} + d^{Oss}\right) = q^{Oss} \Rightarrow \frac{d^{Oss}}{q^{Oss}} = \rho,$$

$$x^{ss} = \kappa,$$

where $\rho$ is the rate of time preference. Using the definitions of $d^O$ and $q^O$ and assuming an internal solution for extraction yields the following steady-state equilibrium condition:

$$-e_s(ss) = \rho \left[p^{ss} - e_x(ss)\right].$$

Using the functional form for extraction costs, $e = \psi \left(\frac{x_t}{s_t}\right)^{\gamma} x_t$, the above condition becomes:

$$\gamma\psi \left(\frac{\kappa}{s}\right)^{1+\gamma} = \rho \left[p^{ss} - (1+\gamma)\psi \left(\frac{\kappa}{s}\right)^{\gamma}\right]$$

which can be rewritten as:

$$\psi \left(\frac{\kappa}{s}\right)^{\gamma} \left[\gamma \left(\frac{\kappa}{s}\right) + \rho(1+\gamma)\right] = \rho p^{ss}. \tag{G19}$$

The steady state oil reserves $s^{ss}$ is the value of $s$ that solves the above equation. Since the left-hand-side is a decreasing, convex function of $s$, the condition determines a unique value of $s^{ss}$ that rises as $p^{ss}$ falls. Hence, a permanent decline in oil prices causes a permanent increase in oil reserves.

In the stochastic version of this setup, the planner uses oil reserves for self insurance, since there are no state-contingent claims to hedge oil-price shocks and no credit market of non-state-contingent international bonds. The Euler equation becomes: $u'(c_t) = \beta E_t \left[R^O_{t+1} u'(c_{t+1})\right]$. The asset price of oil is still positive and given by $q^O_t = E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} d^O_s\right]$. Because of self insurance, the long-run average of reserves in this economy will be larger than $s^{ss}$ (i.e., the planner builds a buffer stock of precautionary savings in the form of oil reserves).

In Appendix I, we present the recursive formulation of this financial autarky setup and derive key properties of the associated dynamic programming problem. In particular, we show that non-negativity of oil profits and a coefficient $\psi$ in the extraction cost function larger than the largest realization of $p$ guarantee that the decision rule on reserves $s'(s, p, y)$ is increasing is $s$ and that the lower bound on $s_{t+1}$ (i.e., the upper bound on $x_t$) is never binding.

## G.2 Exogenous $q$

Consider next the small-open-economy case with a constant, world-determined real interest rate such that $R^b(s_{t+1}, b_{t+1}) = R^*$. Without uncertainty, the Euler equations for bonds and reserves yield the following no-arbitrage condition for the real returns on bonds and oil:

$$R^o_{t+1} = \frac{u'(c_t)}{\beta u'(c_{t+1})} = R^*. \tag{G20}$$

Using the law of motion of reserves and the definitions of the asset price of oil and oil dividends, this no-arbitrage condition yields the following condition (assuming an internal solution for $x_t$ for simplicity):

$$\frac{p_{t+1} - e_x\left(s_{t+1} - s_{t+2}, s_{t+1}\right) - e_s\left(s_{t+1} - s_{t+2}, s_{t+1}\right)}{p_t - e_x\left(s_t - s_{t+1}, s_t\right)} = R^*. \tag{G21}$$

This is a second-order difference equation in $s$ that pins down the optimal decisions for $\{x_t, s_{t+1}\}_{t=0}^{\infty}$ as functions of oil prices and reserves only (and the parameter values of the extraction cost function and $R^*$). Hence, this setup is akin to the deterministic small-open-economy model with capital accumulation in which there is "Fisherian separation" of the investment and production decisions from the consumption and savings plans. Here, the same happens with the optimal plans for oil extraction and accumulation of oil reserves: they are determined independently of those for consumption and debt.

Assuming $\beta R^* = 1$, consumption is perfectly smooth for all $t$, while reserves and extraction follow the dynamics governed by the above second-order difference equation. The sovereign adjusts bond holdings as necessary so that consumption is perfectly smooth while extraction follows its transitional dynamics towards its steady state. This determines the present value of oil income net of extraction costs, and given that the perfectly smooth level of consumption is determined so as to satisfy the intertemporal resource constraint (i.e. the present value of constant consumption equals the present value of oil plus non-oil GDP plus initial bond holdings).

Since $e(\cdot)$ is increasing in $x_t$ and decreasing in $s_t$, the above condition implies that, when $p_{t+1}$ rises relative to $p_t$, the planner reallocates extraction from $t$ to $t+1$ (i.e. increases the accumulation of reserves at $t$). This is a key incentive that is also a work in the model with default risk, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As we demonstrate in Appendix G, default incentives strengthen when oil prices are low and the set of pairs of income and oil prices at which default is preferable shrinks as reserves grow.

This model's deterministic steady state is analogous to the one of the financial autarky case, except that the net world real interest rate $r^* = R^* - 1$ replaces the rate of time preference. Hence, the condition pinning down the deterministic steady state of reserves becomes:

$$\psi\left(\frac{\kappa}{s}\right)^{\gamma}\left[\gamma\left(\frac{\kappa}{s}\right) + r^*(1+\gamma)\right] = r^* p^{ss}.$$

As in the case of financial autarky, there is a unique deterministic steady state for $s^{ss}$ and it increases as the steady-state price of oil falls.

The stochastic version of the model yields a standard equity-premium expression for the excess return on oil:

$$E_t\left[R^O_{t+1}\right] - R^*_{t+1} = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), R^O_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]},$$

This is also analogous to the expression that a standard small-open-economy RBC model would yield. Bonds are ued for self-insurance (i.e., borrowing incentives are weakened by the precautionary savings motive) and extraction and reserves play the role of investment and capital. The asset price of oil is again positive and is now given by $q^O_t = E_t\left[\sum_{s=t+1}^{\infty}(R^*)^{-(s-t)}d^O_s\right]$. Fisherian separation does not hold strictly, because the excess return on oil depends on the marginal utility of consumption, but it holds approximately because equity premia in this class of models are small (as is typical of standard consumption asset pricing models). Hence, the asset price of oil is approximately independent of consumption and savings decisions.

### G.3 Endogenous $q$

The third case takes into account the ad-hoc bond pricing function. Without uncertainty, the Euler equations for bonds and reserves (eqs. (G10) and (G11)) imply the following no-arbitrage condition:

$$R^O_{t+1} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right)\left[1 + \frac{q_s\left(s_{t+1}, b_{t+1}\right)b_{t+1}}{q^O_t}\right]. \tag{G22}$$

Using the alternative definition of the returns on oil that imputes the effect of reserves on bond prices, and since the planner arbitrages returns on bonds and oils against the intertemporal marginal rate of substitution, we obtain that:

$$\tilde{R}^O_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right). \tag{G23}$$

It follows from these conditions that this model's deterministic steady state is pinned down by a two-equation nonlinear system in $(b^{ss}, s^{ss})$ formed by $\tilde{R}^O\left(s^{ss}, b^{ss}\right) = 1/\beta$ and $R^b\left(s^{ss}, b^{ss}\right) = 1/\beta$. The asset price of oil is still positive in this economy, and is simply determined by the deterministic version of eq. (G14).

The conditions that characterize the equilibrium of this economy under uncertainty are the ones provided in the generic characterization of the setup. Equations (G10), (G11), (G14) and (G15) are, respectively, the Euler equations for bonds and reserves, the oil asset-pricing equation and the oil risk premium. This economy is akin to the RBC-like case where there is no default risk, except that in this case the interest rate rises as bonds and/or reserves fall, whereas in the RBC case it remains constant. It also differs in that the planner chooses bonds and reserves internalizing how those choices affect the price of bonds and thus the cost of borrowing, but all of this is done under commitment to repay. Intuitively, it is as if the government acts as a monopolist when it sells its debt.

# H Theoretical Results on Debt, Reserves & Country Risk

This Section of the Appendix derives theoretical results about how country risk and default incentives are affected by the debt position, oil reserves and the realizations of non-oil GDP and oil prices. These results show the extent to which existing results from the sovereign default literature extend to the model we proposed, and provide insights about how oil reserves and oil prices interact with country risk and default incentives. Extending the analysis of standard default models is not straightforward, because in those models the default payoff is exogenous to the sovereign's actions, whereas in our model it depends on the sovereign's optimal plans for oil reserves. As we explain below, this is particularly important for deriving results related to how default sets respond to oil reserves, what contracts are feasible under repayment when default is possible, and how shocks to $y$ and $p$ affect default incentives.

Since some of the propositions rely on conjectures, impose parameter restrictions (i.i.d shocks, $\lambda = 0$, $\hat{p} = p$), and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions in the calibrated model. As reported in Table H1, all the propositions and conjectures hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 which holds in 98 percent of the corresponding evaluations.

Table H1: Validation of Propositions and Conjectures in the Baseline Model

| **Conjecture or Proposition** | **Case** | **Holds in %** | **Max. Error** |
|---|---|---|---|
| **Conjecture 1*** | Repayment | 100 | |
| | Default | 100 | |
| **Conjecture 2** | | 98 | $\tilde{c}^{nd}\left(b, s^2, p, y\right) - \tilde{c}^{nd}\left(b, s^1, p, y\right) = -0.2$ |
| **Conjecture 3** | | 100 | |
| **Proposition 1** | | 100 | |
| **Proposition 2** | $s$ | 100 | |
| | $s'$ | 100 | |
| **Proposition 3** | Repayment | 100 | |
| | Default | 100 | |
| **Proposition 4** | | 100 | |
| **Proposition 5** | | 100 | |
| **Proposition 6** | | 100 | |

*Note:* *This conjecture is evaluated computing oil asset prices as the expected present value of dividends

We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 5 and 6 (see Table H2).[5] Profits are strictly positive for all optimal decision rules of $s'$ under repayment and default. The trade balance conditions of Propositions 5 and 6 hold 97 and 100 percent of all model evaluations, respectively. Removing the trade balance conditions, the main results of those propositions, namely that default incentives strengthen at lower $y$ (Proposition 5) or lower $p$ (Proposition 6), both hold 100 percent of the model evaluations. Thus, in our calibrated numerical solution, lower oil prices and lower non-oil GDP *always* strengthen default incentives.

[5]We also checked whether the boundary conditions for $x$ (or $s'$) bind and found that they are never binding.

Table H2: Additional Conditions on the Validation of Propositions and Conjectures in the Baseline Model

| Condition or Proposition | Case | Validation | Holds in % | Max. Error |
|---|---|---|---|---|
| **Trade balance condition** | Proposition 5 | $tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)$ for $y_2 \in D\left(b, s\right)$ | 97 | $-0.05^*$ |
| | Proposition 6 | $tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)$ for $p_2 \in D\left(b, s\right)$ | 100 | |
| **Reserves choice condition** | Proposition 6 | $s^1 \leq \tilde{s}^2$ | 100 | |
| **Proposition 5** | Without trade balance condition | For all $y_1 < y_2$, and $y_2 \in D\left(b, s\right)$ then $y_1 \in D\left(b, s\right)$ | 100 | |
| **Proposition 6** | Without trade balance condition or $s^1 \leq \tilde{s}^2$ | For all $p_1 < p_2$, and $p_2 \in D\left(b, s\right)$ then $p_1 \in D\left(b, s\right)$ | 100 | |
| **Profits in optimal decisions** | Repayment | $M^{nd}\left(s'^{nd}\left(s, p, y\right), s, p, y\right) > 0$ | 100 | |
| | Default | $M^{d}\left(s'^{d}\left(s, p, y\right), s, p, y\right) > 0$ | 100 | |
| $s^{nd}\left(b, s, p, y\right)$ **boundaries hit** | Lower bound | $s^{nd}\left(b, s, p, y\right) = (s + \kappa) - s(p/\psi)^{(1/\gamma)}$ | 0 | |
| | Upper bound | $s^{nd}\left(b, s, p, y\right) = s + k$ | 0 | |
| $s^{d}\left(s, p, y\right)$ **boundaries hit** | Lower bound | $s^{d}\left(b, s, p, y\right) = (s + \kappa) - s(p/\psi)^{(1/\gamma)}$ | 0 | |
| | Upper bound | $s^{d}\left(b, s, p, y\right) = s + k$ | 0 | |

*Note:* $^*$The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)\right]$

$^{**}$The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)\right]$

$^{**}$The Max. Error is computed as $s^1 - \tilde{s}^2$

For the analysis that follows, we define these functions:

(a) Profits from oil extraction under repayment and default (using the law of motion of reserves to express oil extraction as a function $x(s', s) = s - s' + \kappa$):

$$M^{nd}(s', s, p) \equiv px(s', s) - e(x(s', s), s), \quad M^{d}(s', s, p) \equiv h(p)x(s', s) - e(x(s', s), s).$$

(b) Asset prices of oil under repayment and default:[6]

$$q^{Ond}(s', s, p) \equiv p - e_x(x(s', s), s), \quad q^{Od}(s', s, p) \equiv h(p) - e_x(x(s', s), s).$$

(c) Trade balance under repayment:

$$tb(b', s', b, y, p) \equiv q\left(b', s', y, p\right) b' - b.$$

(d) Consumption under repayment and default:

$c^{nd}(b', s', b, s, y, p) \equiv y - A + M^{nd}(s', s, p) - tb(b', s', b, y, p), \quad c^{d}(s', s, y, p) \equiv y - A + M^{d}(s', s, p)$.

Next, we postulate three conjectures that are used later to prove some of of the propositions in this Appendix:

[6] In Appendix F, we showed that in a model without default risk $p - e_x(x(s', s), s)$ is equal to the asset price of oil (i.e., the expected present value of oil dividends discounted with the sovereign's stochastic discount factors) for internal solutions of $x$ and it is always positive.

**Conjecture 1.** ***Asset prices of oil are positive under repayment and default.***

*$q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ for all $p$, $s \in [\underline{s}, \overline{s}] = \{s : \underline{s} \leq s \leq \overline{s}\}$, and $s'$ in the interval $(s + \kappa) - s(p/\psi)^{(1/\gamma)} \leq s' \leq (s + \kappa)$, where $s' \geq s + \kappa - s(p/\psi)^{(1/\gamma)}$ is implied by the upper bound of $x$ above which profits are negative and $s' \leq s + \kappa$ is the upper bound of reserves if $x = 0$.*

Appendix F shows that this conjecture is an equilibrium outcome for three variants of the model in which the sovereign can commit to repay (i.e., financial autarky and a small open economy facing either a constant real interest rate or an exogenous interest rate function with the qualitative features of the equilibrium interest rate of a model with default). This is because the equilibrium asset price of oil equals the expected present value of the stream of (non-negative) oil dividends discounted with the stochastic discount factor of the sovereign. Assuming $\lambda = 0$, it can also be proven that $q^{Od}(\cdot) > 0$ is an equilibrium outcome in the model with default, because with permanent exclusion the planner's dynamic programming problem is the same as that with commitment to repay under financial autarky.[7]

**Conjecture 2.** ***If default is possible for some state $(b, \tilde{s}, y, p)$, the optimal consumption choice under repayment is nondecreasing in $s$ in the interval $\underline{s} \leq s \leq \tilde{s} \leq \overline{s}$.***

*For all $s^1, s^2 \in [\underline{s}, \tilde{s}]$ and $s^1 \leq s^2$, $\hat{c}^{nd}(b, s^2, y, p) \geq \hat{c}^{nd}(b, s^1, y, p)$, where optimal consumption under repayment is: $\hat{c}^{nd}(b, s, y, p) \equiv y - A + M^{nd}(s'(b, s, y, p), s, y, p) - tb(b'(b, s, y, p), s'(b, s, y, p), b, y, p)$, and $b'(b, s, y, p), s'(b, s, y, p)$ are the bonds and reserves decision rules under repayment, respectively.*

This conjecture is also an equilibrium outcome if the sovereign is committed to repay. It is a standard result that follows from consumption being increasing in wealth but proving this property is not straightforward in the model with default, because it requires properties of decision rules under repayment that are difficult to establish since the optimization problem under repayment retains the option to default in the future and is not differentiable.

**Conjecture 3.** ***If default on outstanding debt is optimal at a given level of existing reserves for some realizations of income and oil prices, all the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits between repayment and default.***

*If for some $(b, s)$ the default set is non-empty $D(b, s) \neq \oslash$, then for $(y, p) \in D(b, s)$ there are no contracts $\{q(b', s', y, p), b', s'\}$ available such that $tb(b', s', b, y, p) < M^{nd}(s', s, p) - M^d(s^d(s, y, p), s, p)$,*

[7] We showed in Appendix F that under financial autarky and assuming an internal solution for $x$, $q_t^O = E_t \sum_{j=1}^{\infty} \beta^j u'(t + j)/u'(t)[-e_s(t + j)]$. This corresponds to $q^{Od}(s', s, p)$ if the probability of re-entry is zero.

*where $s^d(s, y, p)$ is the optimal choice of reserves under default.*

This conjecture is related to Proposition 2 in Arellano (2008). She shows that, assuming i.i.d. shocks, $\lambda = 0$, and no default income costs, if the default set is non-empty for $b$ then there are no contracts $\{q(b'), b'\}$ under repayment that can yield more net resources for current consumption than the resources available under default. Under default, resources are determined by the *exogenous* realization of $y$, which is the same under repayment, so this result implies also that there are no contracts that can yield a trade deficit. In our model, however, the debt contracts may need to entail a trade surplus in order to match the property that they cannot generate more net resources for current consumption than what the *endogenous* choice of oil profits generates under default. This is clearer if we consider that Conjecture 3 implies: $tb(b', s', b, y, p) \geq M^{nd}(s', s, p) - M^d(s^d(s, y, p), s, p)$. If profits under repayment are larger than under default (which is the case if a lower $s'$ is chosen under repayment, since Proposition 2 below shows that profits are decreasing in $s'$), all available debt contracts generate trade surpluses at least as large as the amount by which oil profits under repayment exceed those under default. A zero trade balance is not sufficient to guarantee that there are fewer net resources for consumption under repayment.[8]

**Proposition 1.** ***The repayment payoff is non-decreasing in $b$ and default sets shrink as $b$ rises (i.e. grow as debt rises).***

*For all $b^1 \leq b^2$, $v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$. Moreover, if default is optimal for $b^2$ ($d(b^2, s, y, p) = 1$) for some states $(s, y, p)$ then default is optimal for $b^1$ for the same states $(s, y, p)$ (i.e. $D(b^2, s) \subseteq D(b^1, s)$ and $d(b^1, s, y, p) = 1$)*

*Proof.* This proof follows Arellano (2008).

1. From the definition of $D(\cdot)$ and $d(b^2, s, y, p) = 1$ it follows that $v^d(s, y, p) \geq v^{nd}(b^2, s, y, p)$ $\forall \{y, p\} \in D(b^2, s)$, hence:

$$v^d(s, y, p) \geq v^{nd}(b^2, s, y, p) \geq u\left(c^{nd}(b', s', b^2, s, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \quad \forall (b', s')$$

[8] We can show that Conjecture 3 holds as a proposition under the sufficiency condition that, if the default set is not empty for a pair $(b, s)$, there are no available debt contracts under repayment with associated choices of oil reserves that are smaller than the reserves chosen under default (i.e., the planner cannot generate more resources by setting $s'$ lower in repayment than in default). However, this condition fails in the majority of the state space of the numerical solution with the baseline calibration.

2. It follows that, since $b^1 \leq b^2$ implies $c^{nd}(b', s', b^2, s, y, p) \geq c^{nd}(b', s', b^1, s, y, p)$, the continuation values for $b^1 \leq b^2$ satisfy:

$$u\left(c^{nd}(b', s', b^2, s, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \geq u\left(c^{nd}(b', s', b^1, s, y, p))\right) + \beta E\left[V(b', s', y', p')\right],$$

for all $(b', s')$, which implies that $v^{nd}(b, s, y, p)$ is nondecreasing in $b$.

3. It follows from 1. and 2. that $v^d(s, y, p) \geq v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$, hence $v^d(s, y, p) \geq v^{nd}(b^1, s, y, p)$ which implies $\{y, p\} \in D(b^1, s)$ and thus $d(b^1, s, y, p) = 1$.

□

**Proposition 2.** ***If asset prices of oil are positive, oil profits are increasing in $s$, for given $s'$, and decreasing in $s'$, for given $s$.***

*Given Conjecture 1, oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M_s^{nd}(\cdot), M_s^d(\cdot) > 0$, and decreasing in $s' \in [s+\kappa-s(p/\psi)^{(1/\gamma)}, s+\kappa]$, namely $M_{s'}^{nd}(\cdot), M_{s'}^d(\cdot) < 0$.*

*Proof.* We show first that profits are increasing in $s$, and then that they are decreasing in $s'$.

1. The derivatives of oil profits with respect to $s$ under repayment and default are $M_s^{nd}(\cdot) = p - e_x(x(s', s), s) - e_s(x(s', s), s)$ and $M_s^d(\cdot) = h(p) - e_x(x(s', s), s) - e_s(x(s', s), s)$.

2. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s)$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s)$ respectively.

3. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ and $e_s(x(s', s), s) < 0$ for $s \in [\underline{s}, \overline{s}]$, it follows that $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s) > 0$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s) > 0$.

4. The derivatives of oil profits with respect to $s'$ under repayment and default are $M_{s'}^{nd}(\cdot) = -p + e_x(x(s', s), s))$ and $M_s^d(\cdot) = -h(p) + e_x(x(s', s), s)$.

5. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = -q^{Ond}(s', s, p))$ and $M_s^d(\cdot) = -q^{Od}(s', s, p)$ respectively.

6. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$, it follows that $M_{s'}^{nd}(\cdot) = -q^{Ond}(s', s, p) < 0$ and $M_{s'}^d(\cdot) = -q^{Od}(s', s, p) < 0$.

$\square$

**Proposition 3.** ***The default and repayment payoffs are non-decreasing in*** $s$***.***

*For all* $s^1, s^2 \in [\underline{s}, \overline{s}]$ *and* $s^1 \leq s^2$*,* $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ *and* $v^d(s^2, y, p) \geq v^d(s^1, y, p)$*.*

*Proof.* This proof uses the consumption functions $c^{nd}(b', s', b, s, y, p), c^d(s', s, y, p)$.

1. Since $s^1 \leq s^2$, the result that oil profits are increasing in $s$ (Proposition 2) and the definitions of the consumption functions imply that $c^{nd}(b', s', b, s^2, y, p) \geq c^{nd}(b', s', b, s^1, y, p)$ and $c^d(s', s^2, y, p) \geq c^d(s', s^1, y, p)$ for all $(b', s')$. Hence, the continuation values for $s^1 \leq s^2$ satisfy:

$$\begin{aligned} v^{nd}(b, s^2, y, p) &\geq u\left(c^{nd}(b', s', b, s^2, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \\ &\geq u\left(c^{nd}(b', s', b, s^1, y, p))\right) + \beta E\left[V(b', s', y', p')\right], \end{aligned}$$

   for all $(b', s')$, which implies that $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$. Hence, $v^{nd}(b, s, y, p)$ is nondecreasing in $s$.

2. Similarly, the default payoffs satisfy:

$$\begin{aligned} v^d(s^2, y, p) &\geq u\left(c^d(s', s^2, y, p)\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda)v^d(s', y', p')\right] \\ &\geq u\left(c^d(s', s^1, y, p))\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda)v^d(s', y', p')\right], \end{aligned}$$

   for all $s'$, which implies that $v^d(s^2, y, p) \geq v^d(s^1, y, p)$. Hence, $v^d(s, y, p)$ is nondecreasing in $s$.

$\square$

**Proposition 4.** ***Default sets shrink as*** $s$ ***rises (i.e. grow as reserves fall).***

*Assume* $\hat{p} = p$ *and* $\lambda = 0$ *for simplicity. For all* $s^1, s^2 \in [\underline{s}, \overline{s}]$ *and* $s^1 \leq s^2$*, if default is optimal for* $s^2$ $(d(b, s^2, y, p) = 1)$ *for some states* $(b, y, p)$*, then default is optimal for* $s^1$ *for the same states* $(b, y, p)$ *(i.e.* $D(b, s^2) \subseteq D(b, s^1)$ *and* $d(b, s^1, y, p) = 1$*).*

*Proof.* We show first that this proposition is valid if the decision rules for oil reserves under default and repayment are such that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$, and then we show that this condition holds under Conjecture 2.[9] The proof also requires Conjectures 1 and 3.

1. Since $d(b, s^2, y, p) = 1$ implies $v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq 0$ and both $v^{nd}(b, s, y, p)$ and $v^d(s, y, p)$ are nondecreasing in $s$, in order for $v^d(s^1, y, p) - v^{nd}(b, s^1, y, p) \geq 0$ (i.e. $d(b, s^1, y, p) = 1$), we need to show that when $s$ falls, the default payoff falls as much or less than the repayment payoff: $v^d(s^2, y, p) - v^d(s^1, y, p) \leq v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p)$.

2. Using the definition of $v^d(b, s, p)$ and since $s^d(s^2, y, p)$ is the optimal reserves choice under default when $s = s^2$, it follows that the difference $v^d(s^2, y, p) - v^d(s^1, y, p)$ satisfies this condition:

$$v^d(s^2, y, p) - v^d(s^1, y, p) \leq$$
$$u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1 - \lambda) v^d(s^d(s^2, y, p), y', p')\right]$$
$$-u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') - (1 - \lambda) v^d(s^d(s^2, y, p), y', p')\right]$$

which reduces to:

$$v^d(s^2, y, p) - v^d(s^1, y, p) \leq u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) - u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right)$$

3. Using the definition of $v^{nd}(b, s, p)$ and since $b'(b, s^1, y, p), s'(b, s^1, y, p)$ are the bonds and reserves decision rules under repayment when reserves are $s = s^1$, respectively, it follows that the difference $v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p)$ satisfies this condition:

$$v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p) \geq$$
$$u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^2, y, p)\right) + \beta E\left[V(b'(b, s^1, y, p), s'(b, s^1, y, p), y', p')\right]$$
$$-u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right) + \beta E\left[V(b'(b, s^1, y, p), s'(b, s^1, y, p), y', p')\right]$$

which reduces to:

$$v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p) \geq$$
$$u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^2, y, p)\right) - u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right)$$

[9] Conjecture 2 could be replaced with the assumption that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ and the last step of the proof would be unnecessary, but Conjecture 2 is more reasonable because it states a familiar property of consumption decision rules (i.e. that they are increasing in wealth) and only with respect to consumption under repayment, whereas $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ refers to decision rules for reserves under default with higher $s$ v. repayment with lower $s$.

4. The results in 3. and 4. imply the following sufficiency condition for $v^d(s^2,y,p) - v^d(s^1,y,p) \le v^{nd}(b,s^2,y,p) - v^{nd}(b,s^1,y,p)$:

$$u\left(c^d(s^d(s^2,y,p),s^2,y,p)\right) - u\left(c^d(s^d(s^2,y,p),s^1,y,p)\right) \le$$
$$u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^2,y,p)\right) - u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^1,y,p)\right),$$

which, using the definitions of $c^{nd}(\cdot)$ and $c^d(\cdot)$ and noting that since $\hat{p}=p$ we can write the profit functions as $M^d(\cdot)=M^{nd}(\cdot)=M(\cdot)$, can be rearranged as follows:

$$\begin{aligned} & u\left(y - A + M(s^d(s^2,y,p),s^2,p)\right) \\ & \quad - u\left(y - A + M(s'(b,s^1,y,p),s^2,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right) \\ & \qquad \le u\left(y - A + M(s^d(s^2,y,p),s^1,p)\right) \\ & \quad - u\left(y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p))\right), \end{aligned}$$

and using this notation $\tilde{y}^2 \equiv y - A + M(s^d(s^2,y,p),s^2,p)$, $\tilde{y}^1 \equiv y - A + M(s^d(s^2,y,p),s^1,p)$, $z^2 = M(s'(b,s^1,y,p),s^2,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p) - M(s^d(s^2,y,p),s^2,p)$, $z^1 = M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)) - M(s^d(s^2,y,p),s^1,p)$ it can be re-written as:

$$u\left(\tilde{y}^2\right) - u\left(\tilde{y}^2 + z^2\right) \le u\left(\tilde{y}^1\right) - u\left(\tilde{y}^1 + z^1\right),$$

5. The strict concavity of $u(\cdot)$ implies that the above condition holds if we can show that $\tilde{y}^2 > \tilde{y}^1$ and $z^1 \le z^2 \le 0$. Since $M_s(\cdot) > 0$ as shown in Proposition 2, it follows that $\tilde{y}^2 > \tilde{y}^1$. Conjecture 3 implies that if the default set for $(b,s)$ is not empty, then all the contracts available under repayment are such that $M(s',s,p) - tb(b',s',b,y,p) - M(s^d(s,y,p),s,p) \le 0$, therefore $z^1, z^2 \le 0$. Hence, $z^1 \le z^2 \le 0$ holds if

$$M(s^d(s^2,y,p),s^2,p) - M(s^d(s^2,y,p),s^1,p) \le M(s'(b,s^1,y,p),s^2,p) - M(s'(b,s^1,y,p),s^1,p).$$

Since $M_{ss'}(\cdot) \ge 0$, it follows that the above condition holds if the reserves decision rules under default and repayment are such that $s^d(s^2,y,p) \le s'(b,s^1,y,p)$.[10]

[10]Given the functional form of $e(x,s)$, it is straightforward to show that $M^{nd}_{ss'}(\cdot) = M^d_{ss'}(\cdot) = e_x(\cdot)\gamma(s' - \kappa)/(xs)$. Moreover, we show in Appendix I that under financial autarky (or under default with permanent exclusion), the optimal reserves decision rule is increasing in reserves if $p^{max} < \psi$ (i.e. if the highest realization of oil prices is smaller than the coefficient $\psi$ of the extraction costs function). Hence, $min(s'-x) = s[1-(p/\psi)^{1/\gamma}]$ and therefore $M^{nd}_{ss'}(\cdot) = M^d_{ss'}(\cdot) > 0$.

6. Finally, we show that a sufficiency condition for $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ to hold is that $\hat{c}^{nd}(b, s^2, y, p) \geq \hat{c}^{nd}(b, s^1, y, p)$, which holds because of Conjecture 2. To show this, note first that because of Conjecture 3 (if the default set for $(b, s^1)$ is not empty) $tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)) \geq M(s'(b, s^1, y, p), s^1, p) - M(s^d(s^1, y, p), s^1, p)$, and hence $M(s^d(s^1, y, p), s^1, p) \geq M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)$. Moreover, in the optimization problem under full financial autarky of Appendix F (which is the same as the default problem since $\lambda = 0$) $dM(s', s, p)/ds > 0$.[11] Hence, these two result imply that:

$$
\begin{aligned}
M(s^d(s^2, y, p), s^2, p) &> M(s^d(s^1, y, p), s^1, p) \\
&\geq M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p),
\end{aligned}
$$

therefore:

$$
\begin{aligned}
&y - A + M(s^d(s^2, y, p), s^2, p) \geq \\
&\qquad y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)
\end{aligned}
$$

Since $u(c)$ is increasing in $c$:

$$
\begin{aligned}
&u\Big(y - A + M(s^d(s^2, y, p), s^2, p)\Big) \geq \\
&\qquad u\left(y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)\right)
\end{aligned}
$$

Add $\beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right]$ to both sides of the above expression and simplify using the definition of $v^d(s^2, y, p)$:

$$
\begin{aligned}
v^d(s^2, y, p) \geq\ & u\left(y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)\right) \\
&+ \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right]
\end{aligned}
$$

Subtracting $v^{nd}(b, s^2, y, p)$ from both sides yields:

$$
\begin{aligned}
&v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq \\
&\quad u\left(y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)\right) \\
&\quad - v^{nd}(b, s^2, y, p) + \beta E\left[\lambda V(0, s^d(s^2, p), y', p') + (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right],
\end{aligned}
$$

[11] From the definition of $M(s', s, p)$ it follows that $dM/ds = q^{Od}(s', s, p)[1 - \partial s^d(\cdot)/\partial s] - e_s(\cdot) > 0$, because $q^O d(s', s, p) > 0$, $e_s(\cdot) < 0$ and we conjecture that $\partial s^d(\cdot)/\delta s < 1$ for local stability (Appendix I proves that $\partial s^d(\cdot)/\delta s > 0$).

which using the definitions of $v^{nd}(b, s^2, y, p)$ and $c^{nd}(b', s', b, s, y, p)$ can be written as:

$$\begin{aligned} v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right) \\ - \left[u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) + \beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]\right] \\ + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right], \end{aligned}$$

and rearranging terms in the above expression yields:

$$\begin{aligned} u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) - u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right) \\ + \beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - \lambda V(0, s^d(s^2, y, p), y', p') - (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right] \\ \geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p). \end{aligned}$$

Since $\lambda = 0$, and using the definition of the optimal consumption decision rule $\hat{c}^{nd}(b, s, y, p)$, this expression can be written as:

$$\begin{aligned} u\left(\hat{c}^{nd}(b, s^2, y, p)\right) - u\left(\hat{c}^{nd}(b, s^1, y, p)\right) \\ + \beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - v^d(s^d(s^2, y, p), y', p')\right] \\ \geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p). \end{aligned}$$

This inequality holds because $d(b, s^2, y, p) = 1$ implies that the right-hand-side of this expression is non-positive $(v^{nd}(b, s^2, y, p) - v^d(s^2, y, p) \leq 0)$ while the left-hand-side is non-negative because: (a) Conjecture 2 and the fact that $u(c)$ is increasing in $c$ imply that $u\left(\hat{c}^{nd}(b, s^2, y, p)\right) \geq u\left(\hat{c}^{nd}(b, s^1, y, p)\right)$, and (b) $E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]$ $-E\left[v^d(s^d(s^2, y, p), y', p')\right] \geq 0$ by the definition of $V(\cdot)$.

□

**Proposition 5.** ***If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.***

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $y_1 < y_2$, if $y_2 \in D(b, s)$ and $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ (where $b^1 \equiv b'(b, s, y_1, p)$, $s^1 \equiv s'(b, s, y_1, p)$ are the optimal choices of bonds and reserves under repayment with $y_1$ and $\tilde{s}^2 \equiv s^d(s, y_2, p)$ is the optimal reserves choice under default with $y_2$), then $y_1 \in D(b, s)$.*

*Proof.* This proof aims to extend Proposition 3 in Arellano (2008), but for this model it requires a lower bound condition on the trade balance linked to the optimal decision rules of reserves under repayment with $y_1$ v. under default with $y_2$.

1. If $y_2 \in D(b,s)$ and denoting $b^2 \equiv b'(b,s,y_2,p)$, $s^2 \equiv s'(b,s,y_2,p)$ the optimal choices of bonds and reserves when $y = y_2$ under repayment, it follows that by definition:

$$u\left(y_2 - A + M^d(\widetilde{s^2}, s, p)\right) + \beta E[v^d(\widetilde{s^2}, y', p')] \geq$$
$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$

2. To establish that $y_2 \in D(b,s) \Rightarrow y_1 \in D(b,s)$ it is sufficient to show that, denoting $\tilde{s}^1$ as reserves chosen when $y = y_1$ under default, the following holds: $\text{u}\big(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\big)+$ $\beta E[V(b^2, s^2, y', p')] -$
$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$
$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$

3. Given that $(b^2, s^2)$ maximizes the repayment payoff with $y_2$ and $\tilde{s}^1$ maximizes the default payoff with $y_1$, the following two conditions hold:

$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$
$$\geq \left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]$$

$$u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

4. Using the results in 3., the condition in 2. holds if:

$\left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] -$
$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$
$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$

5. The above expression simplifies to:

$$u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) - u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right)$$
$$\geq u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right),$$

which adding and subtracting $M^d(\tilde{s}^2, s, p)$ inside the argument of the repayment utilities and rearranging yields:

$$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_2 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right)$$
$$\leq u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right),$$

where $z(y_1) \equiv M^{nd}(s^1, s, p) - tb(b^1, s^1, b) - M^d(\tilde{s}^2, s, p)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $y_2 > y_1$ and (c) $z(y_1) < 0$ because of the assumption that $tb(b^1, s^1, b) \geq M^{nd}(s^1, s, p) - M^d(\tilde{s}^2, s, p)$.

□

**Proposition 6.** ***If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall.***

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $p_1 < p_2$ and $p_2 \in D(b, s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where $b^1, s^1$ are the optimal bonds and reserves choices under repayment in state $(b, s, y, p_1)$ and $\tilde{s}^2$ is the optimal reserves choice under default in state $(s, y, p_2)$, then $p_1 \in D(b, s)$.*

*Proof.* This proof follows a similar strategy as that of Proposition 5. Again it requires a lower bound condition on the trade balance, but now linked to the optimal decision rules of reserves under repayment with $p_1$ v. under default with $p_2$, and it also requires optimal reserves under default with $p_2$ to exceed those under repayment with $p_1$.

1. If $p_2 \in D(b, s)$ and denoting $(b^2, s^2)$ and $\tilde{s}^2$ as the optimal choices of bonds and reserves when $p = p_2$ under repayment and default, respectively, it follows that by definition::

$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] \geq$$
$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')],$$

   where the profit functions under default and repayment are the same because $\hat{p} = p$.

2. To establish that $p_2 \in D(b, s) \Rightarrow p_1 \in D(b, s)$ it is sufficient to show that, denoting $(b^1, s^1)$ and $\tilde{s}^1$ as the bonds and reserves chosen when $p = p_1$ under repayment and default, respectively, the following holds:

$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')] -$$
$$\left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$$

3. Given that $(b^2, s^2)$ maximizes the repayment payoff with $p_2$ and $\tilde{s}^1$ maximizes the default payoff with $p_1$, the following two conditions hold:

$$
\begin{aligned}
&u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')] \\
&\qquad \geq \left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]
\end{aligned}
$$

$$
u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]
$$

4. Using the results in 3., the condition in 2. holds if:

$$
\begin{aligned}
&\left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] - \\
&\qquad \left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq \\
&u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]
\end{aligned}
$$

5. The above expression simplifies to:

$$
\begin{aligned}
&u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) - u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) \\
&\qquad \geq u\left(y - A + M(\tilde{s}^2, s, p_2)\right) - u\left(y - A + M(\tilde{s}^2, s, p_1)\right),
\end{aligned}
$$

which adding and subtracting $M(\tilde{s}^2, s, p_2)$ and $M(\tilde{s}^2, s, p_1)$ to the arguments of the repayment utility in the left- and right-hand-sides, respectively,and rearranging yields:

$$
\begin{aligned}
&u\left(y - A + M(\tilde{s}^2, s, p_2)\right) - u\left(y - A + M(\tilde{s}^2, s, p_2) + z(p_2)\right) \\
&\leq u\left(y - A + M(\tilde{s}^2, s, p_1)\right) - u\left(y - A + M(\tilde{s}^2, s, p_1) + z(p_1)\right)
\end{aligned}
$$

where: $z(p_1) = M(s^1, s, p_1) - tb(b^1, s^1, b) - M(\tilde{s}^2, s, p_1)$ and $z(p_2) = M(s^1, s, p_2) - tb(b^1, s^1, b) - M(\tilde{s}^2, s, p_2)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $M(\tilde{s}^2, s, p_2) > M(\tilde{s}^2, s, p_1)$ since profits are increasing in $p$, (c) $z(p_2) \leq 0$ because of the assumption that $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$, and (d) $z(p_1) \leq z(p_2)$ because $s^1 \leq \tilde{s}^2$ (note that $z(p_1) \leq z(p_2) \leftrightarrow M(s^1, s, p_1) - M(\tilde{s}^2, s, p_1) \leq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ or $M(\tilde{s}^2, s, p_2) - M(\tilde{s}^2, s, p_1) \leq M(s^1, s, p_2) - M(s^1, s, p_1)$ and using the functional form of $M(.)$ this yields $(p_2 - p_1)(s - \tilde{s}^2 + \kappa) \leq (s - s^1 + \kappa)(p_2 - p_1)$, which implies that $s^1 \leq \tilde{s}^2$).

$\square$

# I Dynamic Programming Problem under Financial Autarky

The dynamic programming problem of the planner under financial autarky, which corresponds also to the default payoff and decision rules when $\lambda = 0$, can be written as follows:

$$V^d(s,p,y) = \underset{s' \in \Gamma(s)}{max} \left\{ F\left(s,s',p,y\right) + \beta E\left[V^d\left(s',p',y'\right)\right] \right\}$$

$$F\left(s,s',p,y\right) \equiv u\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$

$$\Gamma(s) \equiv \left\{ s' : 0 \leq s' \leq s + \kappa \right\},$$

with first-order condition:

$$[s'] : u_c(t)\left(p - e_x(\cdot)\right) = \beta V^d_{s'}\left(s',p',y'\right)$$

or

$$-F_{s'}\left(s,s',p,y\right) = \beta V^d_{s'}\left(s',p',y'\right).$$

This Appendix shows that the period-payoff $F(s,s',p,y)$ of the above problem satisfies standard properties analogous to those of the textbook neoclassical Ramsey model, with oil reserves taking the place of the capital stock. In particular, we show that $F(s,s',p,y)$ is continuously differentiable in $(s,s')$, strictly increasing (decreasing) in $s$ $(s')$, and strictly concave in $(s,s')$. We also show that the optimal decision rule $s'(s,p,y)$ is increasing in $s$. These properties, together with the assumptions that $F(\cdot)$ is bounded and $\Gamma(s)$ is a nonempty, compact-valued, monotone, and continuous correspondence with a convex graph, ensure that the value function $V^d(\cdot)$ that solves the above Bellman equation exists and the solution is unique, and that $V^d(\cdot)$ is strictly concave, strictly increasing and continuously differentiable.[12] The proofs of these properties are analogous to those of the textbook Ramsey model and therefore are omitted here. Existence and uniqueness follow from the contraction mapping theorem. The proof that $V^d(\cdot)$ is increasing requires $F(\cdot)$ to be increasing and $\Gamma(s)$ to be monotone, the proof that $V^d(\cdot)$ is concave requires $F(\cdot)$ to be concave, and proving the differentiability of $V^d(\cdot)$ requires $F(\cdot)$ to be continuously differentiable and concave.

[12]We also assume a standard, twice-continuously-differentiable, increasing and concave utility function. The CRRA utility function that defines $F(\cdot)$ in the numerical solution satisfies these properties but is unbounded. It can be transformed into a bounded function with a piece-wise truncation at an arbitrary small but positive consumption level (see Suen (2009). "Bounding the CRRA Utility Functions," Working Papers 200902, University of California at Riverside, Department of Economics).

**1.** $F(\cdot)$ **is strictly increasing in** $s$ **(**$F_s(\cdot) > 0$**) and decreasing in** $s'$ **(**$F'_s(\cdot) < 0$**).**

To prove these two properties, recall that $e_s(\cdot) < 0$ and that we showed in the sequential solution of the autarky model of Appendix F that the asset price of oil is positive for internal solutions of $x$, hence $p - e_x(\cdot) > 0$. By differentiating $F(\cdot)$ with respect to $s$ and $s'$ we obtain:

$$F_s(\cdot) = u_c(\cdot)\left(p - e_x(\cdot) - e_s(\cdot)\right) > 0,$$

$$F_{s'}(\cdot) = u_c(\cdot)\left(-p + e_x(\cdot)\right) = -u_c(\cdot)\left(p - e_x(\cdot)\right) < 0.$$

**2.** $F(\cdot)$ **is continuously differentiable.**

To prove that $F(\cdot)$ is continuously differentiable, we need to show that: (a) $F(\cdot)$ is continuous in its domain and (b) $F_s(\cdot)$ and $F_{s'}(\cdot)$ exist and are continuous in their domain. For this proof, consider the above expressions for $F_s(\cdot)$ and $F_{s'}(\cdot)$ and express the extraction cost and its derivatives as functions of $s$ and $s'$ using the law of motion $x = s - s' + \kappa$ as follows:

$$e\left(s', s\right) = \psi \frac{\left(s - s' + \kappa\right)^{1+\gamma}}{s^\gamma}$$

$$e_x\left(s', s\right) = (1+\gamma)\,\psi\left(\frac{s - s' + \kappa}{s}\right)^\gamma = (1+\gamma)\,\psi\left(1 - \frac{\left(s' - k\right)}{s}\right)^\gamma$$

$$e_s\left(s', s\right) = -\gamma\psi\left(\frac{s - s' + \kappa}{s}\right)^{1+\gamma} = -\gamma\psi\left(1 - \frac{\left(s' - k\right)}{s}\right)^{1+\gamma},$$

where $e_x(\cdot)$ and $e_s(\cdot)$ are continuous in the domain given by $0 \leq s' \leq s + k$ and $s > 0$ with the following upper and lower bounds:

$$e_x(0, s) = 0, \quad e_s(0, s) = 0$$

$$e_x(s + k, s) = (1+\gamma)\,\psi\left(\frac{s+k}{s}\right)^\gamma, \quad e_s(s + k, s) = -\gamma\psi\left(\frac{s+k}{s}\right)^{1+\gamma}$$

If in addition, oil profits are required to be non-negative, which is analogous to the non-negativity constraint on consumption (or the Inada condition in $u(c)$) in the texbook Ramsey model, the domain of the cost function and its derivatives requires $px \geq e(\cdot)$. Moreover, if oil revenue is the only income or $y - A \leq 0$, the Inada condition would imply that negative profits are never optimal and profits must always be sufficient to sustain $c > 0$. Using again the law of motion $x = s - s' + \kappa$, we obtain that with non-negative profits the lower bound of $s'$ becomes $s' \geq \kappa + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ instead of $s' > 0$. Hence, the domain of $s'$ becomes $\kappa + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + \kappa$.

The functions:

$$F(\cdot) = u\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$

$$F_s(\cdot) = u_c\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$
$$\times\left[p - (1+\gamma)\psi\left(1 - \frac{(s' - \kappa)}{s}\right)^{\gamma} + \gamma\psi\left(1 - \frac{(s' - \kappa)}{s}\right)^{1+\gamma}\right],$$

$$F_{s'}(\cdot) = -u_c\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)\left[p - (1+\gamma)\psi\left(1 - \frac{(s' - \kappa)}{s}\right)^{\gamma}\right],$$

are continuous and exist in the domain defined by $\kappa + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + \kappa$ and $s > 0$.

**3.** $s'(s, p, y)$ **is increasing in** $s$**.**

From the first-order condition for $s'$, this property requires that $-F_{s'}(\cdot) = u_c(\cdot)(p - e_x(\cdot))$ be decreasing in $s$, since $V^d_{s'}(\cdot)$ is independent of $s$. Thus, we need to show that $\frac{\partial - F_{s'}(\cdot)}{\partial s} < 0$.

$$\frac{\partial - F_{s'}(\cdot)}{\partial s} = \left[p - e_x(\cdot)\right]\left[u_{cc}(\cdot)\{p - e_x(\cdot) - e_s(\cdot)\}\right] + u_c(t)\left\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\right\}.$$

Since $e_s(\cdot) < 0$, $p - e_x(\cdot) > 0$, $u_c(\cdot) > 0$ $u_{cc}(\cdot) < 0$, the above expression is negative if $\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\} < 0$. To determine the sign of this expression, use the functional form $e(x, s) = \psi\frac{x^{1+\gamma}}{s^{\gamma}}$ to show that the derivatives $e_{xx}(\cdot)$ and $e_{xx}(\cdot)$ can be expressed as follows:

$$e_{xx}(x, s) = \gamma(1+\gamma)\psi\frac{x^{\gamma}}{s^{\gamma}}x^{-1} = e_x(\cdot)\gamma x^{-1} > 0,$$

$$e_{xs}(x, s) = -\gamma(1+\gamma)\psi\frac{x^{\gamma}}{s^{\gamma}}s^{-1} = -e_x(\cdot)\gamma s^{-1} < 0.$$

Using these expressions, we obtain:

$$\{-\left[e_{xx}(t) + e_{xs}(t)\right]\} = \left\{-\left[e_x(\cdot)\gamma\left(x^{-1} - s^{-1}\right)\right]\right\} < 0 \quad \text{if } x < s,$$

and using $x = s - s' + \kappa$, the condition $x < s$ implies $s - s' + \kappa < s$ which reduces to:

$$s' > \kappa.$$

Hence, $s'(s,p,y)$ is increasing in $s$ if the choice of reserves always exceeds oil discoveries. Since the non-negativity of profits requires $s' \geq \kappa + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ and existing reserves satisfy $s > 0$, the condition $s' > \kappa$ is implied by the non-negativity of profits if $p^{max} < \psi$ (i.e., $\psi$ is larger than the largest realization of oil prices so that $p/\psi$ is always less than 1). This result also implies that the upper bound on $x$ never binds (since $s'$ is always strictly positive because $s' > \kappa > 0$).

**4. $F(\cdot)$ is strictly concave**

To show that $F(\cdot)$ is strictly concave, let $H(\cdot)$ be the Hessian matrix of $F(\cdot)$ defined as

$$H(\cdot) = \begin{bmatrix} F_{ss}(\cdot) & F_{ss'}(\cdot) \\ F_{s's}(\cdot) & F_{s's'}(\cdot) \end{bmatrix}$$

$F(\cdot)$ is strict concave if $H(\cdot)$ is negative definite. That is

- $F_{ss}(\cdot) < 0$
- $F_{ss}(\cdot) F_{s's'}(\cdot) - F_{ss'}(\cdot) F_{s's}(\cdot) > 0$

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\, u_{cc}(\cdot)\, [p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\, [-e_{xx}(\cdot) - e_{sx}(\cdot) - e_{xs}(\cdot) - e_{ss}(\cdot)]$$

Recall

$$e(x,s) = \psi \frac{x^{1+\gamma}}{s^{\gamma}} e_x(x,s) = (1+\gamma)\,\psi \left(\frac{x}{s}\right)^{\gamma} e_s(x,s) = -\gamma\psi\left(\frac{x}{s}\right)^{1+\gamma}$$

Where

1. $e_{xx}(x,s) = \gamma(1+\gamma)\,\psi \frac{x^{\gamma}}{s^{\gamma}} x^{-1} = e_x(\cdot)\,\gamma x^{-1} > 0$
2. $e_{xs}(x,s) = -\gamma(1+\gamma)\,\psi \frac{x^{\gamma}}{s^{\gamma}} s^{-1} = -e_x(\cdot)\,\gamma s^{-1} < 0$
3. $e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi \left(\frac{x}{s}\right)^{1+\gamma} x^{-1} = e_s(\cdot)(1+\gamma)\,x^{-1}$
4. $e_{ss}(x,s) = \gamma(1+\gamma)\,\psi \left(\frac{x}{s}\right)^{1+\gamma} s^{-1} = -e_s(\cdot)(1+\gamma)\,s^{-1}$

Additionally, from 3. we can obtain:

$$e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{1+\gamma} x^{-1} = -e_x(\cdot)\,\gamma s^{-1}$$

Using $-e_x(\cdot)\,\gamma s^{-1} = e_s(\cdot)\,(1+\gamma)\,x^{-1}$,

$$e_x(\cdot) = -e_s(\cdot)\,\frac{(1+\gamma)}{\gamma}x^{-1}s,$$

1. $e_{xx}(x,s) = e_x(\cdot)\,\gamma x^{-1} = -e_s(\cdot)\,(1+\gamma)\,x^{-2}s$
2. $e_{xs}(x,s) = -e_x(\cdot)\,\gamma s^{-1} = e_s(\cdot)\,(1+\gamma)\,x^{-1}$
3. $e_{sx}(x,s) = e_s(\cdot)\,(1+\gamma)\,x^{-1} = e_{xs}(x,s)$
4. $e_{ss}(x,s) = -e_s(\cdot)\,(1+\gamma)\,s^{-1}$

Then

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\,\{-[e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]\}$$

$$\begin{aligned}F_{ss}(\cdot) &= [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] \\ &\quad + u_c(\cdot)\left\{-\left[\left\{-e_s(\cdot)\,(1+\gamma)\,x^{-2}s\right\} + 2\left\{e_s(\cdot)\,(1+\gamma)\,x^{-1}\right\} + \left\{-e_s(\cdot)\,(1+\gamma)\,s^{-1}\right\}\right]\right\}\end{aligned}$$

$$F_{ss}(\cdot) = \overset{\ominus}{u_{cc}}(\cdot)[p - e_x\overset{\oplus}{(\cdot)} - e_s(\cdot)]^2 + \overset{\oplus}{u_c}(\cdot)\left\{e_s(\cdot)\,\overset{\ominus}{(1+\gamma)}\left[x^{-2}s - 2x^{-1} + s^{-1}\right]\right\}$$

For $F_{ss}(\cdot) < 0$ to hold, $\left[x^{-2}s - 2x^{-1} + s^{-1}\right]$ must be positive

$$\begin{gathered}x^{-2}s - 2x^{-1} + s^{-1} > 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) + \frac{1}{s} > 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) > -\frac{1}{s} \\ \left(\frac{s}{x} - 2\right) > -\frac{x}{s} \\ \left(\frac{s}{x} + \frac{x}{s}\right) > 2 \\ s^2 + x^2 - 2sx > 0\end{gathered}$$

$$(s-x)^2 > 0$$

$$\left(s'-k\right)^2 > 0$$

Which holds in domain of

$$k+s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+k$$

$$s > 0$$

Finally for $F_{ss}\left(\cdot\right)F_{s's'}\left(\cdot\right) - F_{ss'}\left(\cdot\right)F_{s's}\left(\cdot\right) > 0$

$$F_{s'}\left(\cdot\right) = -u_c\left(\cdot\right)\left(p-e_x\left(\cdot\right)\right)$$

$$F_{s's'}\left(\cdot\right) = -\left(-p+e_x\left(\cdot\right)\right)u_{cc}\left(\cdot\right)\left(p-e_x\left(\cdot\right)\right) + \left\{-u_c\left(\cdot\right)\left[-e_{xs'}\left(\cdot\right)\right]\right\}$$

$$F_{s's'}\left(\cdot\right) = -\left(-p+e_x\left(\cdot\right)\right)u_{cc}\left(\cdot\right)\left(p-e_x\left(\cdot\right)\right) + \left\{u_c\left(\cdot\right)\left[e_{xs'}\left(\cdot\right)\right]\right\}$$

$$e_x\left(x,s\right) = \left(1+\gamma\right)\psi\left(\frac{s-s'+k}{s}\right)^{\gamma}$$

$$e_{xs'}\left(\cdot\right) = -\gamma\left(1+\gamma\right)\psi\left(\frac{x}{s}\right)^{\gamma}x^{-1} = -\gamma e_x\left(\cdot\right)x^{-1} = -e_{xx}\left(\cdot\right) < 0$$

$$F_{s's'}\left(\cdot\right) = u_{cc}\left(\cdot\right)\left(p-e_x\left(\cdot\right)\right)^2 - \left\{u_c\left(\cdot\right)\left[e_{xx}\left(\cdot\right)\right]\right\}$$

And

$$F_s\left(\cdot\right) = u_c\left(\cdot\right)\left(p-e_x\left(\cdot\right)-e_s\left(\cdot\right)\right)$$

$$F_{ss'}\left(\cdot\right) = \left[\left(-p+e_x\left(\cdot\right)\right)u_{cc}\left(\cdot\right)\left(p-e_x\left(\cdot\right)-e_s\left(\cdot\right)\right)\right] + \left[u_c\left(\cdot\right)\left(-e_{xs'}\left(\cdot\right)-e_{ss'}\left(\cdot\right)\right)\right]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\,(e_{xs'}(\cdot) + e_{ss'}(\cdot))]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\,(-e_{xx}(\cdot) - e_{sx}(\cdot))]$$

$$F_{ss'}(\cdot) = [-(p - e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] + [u_c(\cdot)\,(e_{xx}(\cdot) + e_{sx}(\cdot))]$$

And

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot)) - u_c(\cdot)\,(-e_{xx}(\cdot) - e_{xs}(\cdot))$$

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot)) + u_c(\cdot)\,(e_{xx}(\cdot) + e_{xs}(\cdot))$$

Let

$$M \equiv [p - e_x(\cdot) - e_s(\cdot)]$$

$$q^o \equiv [p - e_x(\cdot)]$$

$$A \equiv [e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]$$

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Rewriting the system

$$F_{ss}(\cdot) = u_{cc}(\cdot)\, M^2 - u_c(\cdot)\, A$$

$$F_{s's'}(\cdot) = u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B$$

$$F_{ss'}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

$$F_{s's}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

Operating $F_{ss}(\cdot)F_{s's'}(\cdot)$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = \left\{u_{cc}(\cdot)M^2 - u_c(\cdot)A\right\}\left\{u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B\right\}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2 AB$$

And $F_{ss'}(\cdot)F_{s's}(\cdot)$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = \{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}\{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}$$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - 2u_{cc}(\cdot)u_c(\cdot)CMq^o + [u_c(\cdot)]^2 C^2$$

So $F_{ss}(\cdot)F_{s's'}(\cdot) - [F_{ss'}(\cdot)]^2 > 0$

$$\begin{aligned} F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = {} & [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2 AB \\ & - [u_{cc}(\cdot)]^2[Mq^o]^2 + 2u_{cc}(\cdot)u_c(\cdot)CMq^o - [u_c(\cdot)]^2 C^2 > 0 \end{aligned}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = -u_{cc}(\cdot)u_c(\cdot)\left[BM^2 - 2CMq^o + A(q^o)^2\right] + [u_c(\cdot)]^2\left[AB - C^2\right]$$

Replacing $\left[AB - C^2\right]$

$$AB = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)e_{xx}(\cdot) + e_{ss}(\cdot)e_{xx}(\cdot)$$

$$C^2 = [e_{xx}(\cdot)]^2 + 2e_{xx}(\cdot)\, e_{sx}(\cdot) + [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)\, e_{xx}(\cdot) + e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{xx}(\cdot)]^2 - 2e_{xx}(\cdot)\, e_{sx}(\cdot) - [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{sx}(\cdot)]^2$$

Recall

1. $e_{xx}(x, s) = e_x(\cdot)\, \gamma x^{-1} = -e_s(\cdot)\,(1+\gamma)\, x^{-2} s$
2. $e_{xs}(x, s) = -e_x(\cdot)\, \gamma s^{-1} = e_s(\cdot)\,(1+\gamma)\, x^{-1}$
3. $e_{sx}(x, s) = e_s(\cdot)\,(1+\gamma)\, x^{-1} = e_{xs}(x, s)$
4. $e_{ss}(x, s) = -e_s(\cdot)\,(1+\gamma)\, s^{-1}$

$$\left[AB - C^2\right] = \left\{-e_s(\cdot)\,(1+\gamma)\, s^{-1}\right\}\left\{-e_s(\cdot)\,(1+\gamma)\, x^{-2} s\right\} - \left[e_s(\cdot)\,(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2\,(1+\gamma)^2\, x^{-2}\right\} - \left[e_s(\cdot)\,(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2\,(1+\gamma)^2\, x^{-2}\right\} - \left\{[e_s(\cdot)]^2\,(1+\gamma)^2\, x^{-2}\right\}$$

$$\left[AB - C^2\right] = 0$$

So the expression $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot)$ is redefined as,

$$F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) = -u_{cc}(\cdot)\, u_c(\cdot)\left[BM^2 - 2CMq^o + A\,(q^o)^2\right]$$

Then, since $-u_{cc}(\cdot)\, u_c(\cdot) > 0$, $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$ holds if,

$$\left[BM^2 - 2CMq^o + A\left(q^o\right)^2\right] > 0$$

Let

$$Z \equiv \frac{M}{q^o}$$

$$\left[BZ^2 - 2CZ + A\right] > 0$$

Solving the inequality

$$Z > \frac{2C \pm \sqrt{4C^2 - 4AB}}{2B}$$

$$Z > \frac{C \pm 2\sqrt{C^2 - AB}}{B}$$

As we show $AB - C^2 = 0$

$$Z > \frac{C}{B}$$

Replacing $Z \equiv \frac{M}{q^o}$

$$\frac{M}{q^o} > \frac{C}{B}$$

Since $M = q^o - e_s\left(\cdot\right)$

$$\frac{q^o - e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{C}{B} > \frac{e_s\left(\cdot\right)}{q^o}$$

$$\frac{B-C}{B} > \frac{e_s(\cdot)}{q^o}$$

Since $\frac{e_s(\cdot)}{q^o} < 0$ it is sufficient to show $B - C > 0$

Recall

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Then

$$B - C > 0$$

$$e_{xx}(\cdot) - e_{xx}(\cdot) - e_{sx}(\cdot) > 0$$

$$-e_{sx}(\cdot) > 0$$

Recall $e_{sx}(\cdot) = e_s(\cdot)(1+\gamma)x^{-1}$

$$-e_s(\cdot)(1+\gamma)x^{-1} > 0$$

Since $e_s(\cdot) < 0$, the condition holds in domain of

$$k + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + k$$

$$s > 0$$

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